# MATHEMATICS MADE EASY

FOR CHILDREN WITH VISUAL IMPAIRMENT
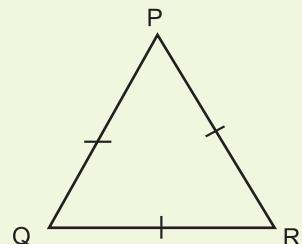
$$\sum x^2 + 9$$
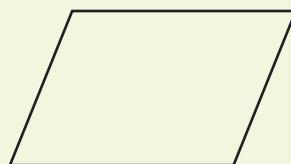
$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$
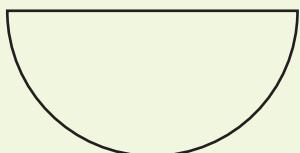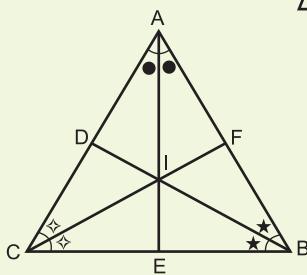
$$\text{Power} = 1 - \beta$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$$

$$\begin{pmatrix} 2 & 3 & -4 \\ 5 & -2 & 5 \end{pmatrix}$$

ICEVI

International Council for Education of People with Visual Impairment

Overbrook School for the Blind

The Nippon Foundation

A Publication of
ON-NET / ICEVI

OVERBROOK SCHOOL FOR THE BLIND
**www.obs.org**

Overbrook School for the Blind offers a variety of programs for children of different ages and abilities. The whole school is geared toward supporting students as they grow and learn.
Since its founding in 1832 our mission has been to provide all of our students, according to their individual needs, with the skills that will give them the greatest opportunity to experience active and fulfilling lives.

OUR VISION
The vision of Overbrook School for the Blind is to be a dynamic and responsive educational organization, providing leadership as a local, national and international resource, inspiring individuals with visual impairments and other challenges to achieve their highest potential.

OUR MISSION STATEMENT
The mission of Overbrook School for the Blind is to develop and deliver education that enhances the options available for persons with visual impairment and other challenges so that they have the greatest opportunity to experience active and fulfilling lives.

First Published : 2005

INTERNATIONAL COUNCIL FOR EDUCATION OF PEOPLE WITH VISUAL IMPAIRMENT (ICEVI)
www.icevi.org

Founded in 1952, the International Council for Education of People with Visual Impairment (ICEVI) is a global association of individuals and organizations that promotes equal access to appropriate education for all visually impaired children and youth so they may achieve their full potential.

ICEVI believes that all children and youth with visual impairment have the same basic human rights as all other persons and should have access to:

- *a full range of educational services of high standard conforming to best practices*

- *teachers and other professionals who are properly trained*

- *appropriate educational materials and methods to meet their needs*

- *parents and family members that understand their needs and support their education and*

- *access to an environment free of barriers, social stigmas and sterotypes*

that allow them to lead a productive life according to their aspirations and capabilities.

THE NIPPON FOUNDATION
www.nippon-foundation.or.jp

The Nippon Foundation was established in 1962, The Foundation believes that all people on earth share a common duty, a mission: of transcending antagonism and overcoming conflict, so that we may instead establish consensus and provide assistance. The Nippon Foundation has dedicated itself to meeting that challenge.

Over the past two decades The Nippon Foundation has devoted considerable attention to the needs of persons with disabilities. Their work in Southeast Asia on behalf of persons with visual impairment has been significant and has resulted in many new programs that are developing leadership in the community of blind persons that is "changing what it means to be blind".

ACKNOWLEDGEMENTS

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DEDICATION

This publication is dedicated to Mr. Yohei Sasakawa and The Nippon Foundation whose belief in the capabilities of persons with visual impairment has allowed them, their parents and their teachers from throughout Southeast Asia to work together to make this project and this publication a reality.

Message from the Director

Overbrook School for the Blind

Overbrook is proud to release this publication with our colleagues at the International Council for Education of People with Visual Impairment (ICEVI) and we hope you will find it of value in strengthening your ability to teach secondary level mathematics concepts to children with visual impairment.

For as long as I have been involved in the education of children with visual impairment, the teaching of mathematics has been a persistent challenge facing educators here in the United States. As you read the introduction to this publications you will note that this is a challenge facing many countries throughout the world. We hope this publication will be of assistance to teachers, parents and students wherever they live in improving mathematics instruction.

Through our International Program here at Overbrook and our collaboration with The Nippon Foundation (Japan) we have been able to work with a very talented and dedicated project development team in creating this resource. Through this team we have been able to work with hundreds of teachers in the Asia region to respond to this challenge and to develop this publication. To all of these individual we express our thanks for their contributions.

We sincerely hope that this publication will assist countless children with visual impairment throughout the world to master secondary level mathematics by providing their teachers and parents with a resource that will improve instructional strategies.

As with all publications, we value your feedback. Both Overbrook and ICEVI look forward to your comments on this publication.

Sincerely,

Bernadette M. Kappen, *Ph.D.*
Director

CONTENTS

Section 5

INTRODUCTION

ON-NET / ICEVI MATHEMATICS PROJECT
THE EVOLUTION OF THIS PUBLICATION

SETTING THE SCENE

In July, 1998 the Overbrook School for the Blind (USA) and The Nippon Foundation (Japan) embarked on a unique and ambitious collaborative effort to improve and expand education and employment opportunities for blind persons in Southeast Asia. The program set out to address needs of blind and low vision persons in eight (8) countries in Southeast Asia (Cambodia, Indonesia, Laos, Malaysia, Myanmar, Philippines, Thailand and Vietnam).

The program was created on the assumption that with: -collaboration at the national and regional levels, -effective use of new technologies and -active involvement of blind individuals, their organizations, teachers and parents would result in improved educational access for blind children and youth and new employment options for educated blind adults. This program, the Overbrook-Nippon Network on Educational Technology, soon became better known by its acronym ON-NET.

In addition to ON-NET national level committees that set the tone and priorities for each country, ON-NET also created a Regional Advisory Committee whose task was to identify challenges that were: -common to all or most of the countries in the region and -might be most effectively addressed through a region strategy.

At the first ON-NET Regional Advisory Committee meeting in December, 2001 the issue of weak instruction in mathematics for blind children was identified as one of several priorities that might best be tackled through a well planned regional strategy. This same group noted that poor instruction in the area was a region-wide weakness and that it was placing

significant impediments in the career path of otherwise well educated blind persons who wanted to pursue careers in the areas of science and technology.

As is the case with most challenges, it is always easier to identify a problem than it is to create a solution. However, in January, 2003 a group of teachers, blind individuals, teacher trainers and members of the ON-NET Regional Advisory Group met in Bangkok and agreed that a plan to prepare a small but very carefully selected group of Master Trainers from throughout the region was something worth doing. It was further agreed that while mathematics instruction at all levels was weak; our focus should be on improving secondary level mathematics instruction for the blind.

In May, 2003 a group of experts met at the ON-NET Regional Center at Ratchasuda College of Mahidol University to work out the details of putting together an expert team to conduct the Master Trainers program and to a comprehensive mathematical package that could be field-tested and then used by the Master Trainers to train others. This publication is the result of that effort over the past three years.

At the Bangkok planning meeting, I invited Dr. M.N.G. Mani, Secretary General, ICEVI and Ms. Aree Plernchaivanich, Director, Region 7 Special Education Center, Phitsanuloke, Thailand to co-chair a Math Task Force that would develop the training program and the training materials. Both Dr. Mani and Ms. Plernchaivanich are experienced educators of children with visual impairment and mathematics teachers. Mr. G.R. Ramesh, Lecturer in Special Education, International Human Resource Development Center for the Disabled, Ramakrishna Mission Vidalaya, Coimbatore, India was later co-opted to the Task Force.

There was unanimous agreement that developing a learning package for teaching mathematics would not only facilitate improved ability of special educators teaching mathematics to blind children, but the materials would also be useful for general classroom teachers as well.

Next, the group tackled the question of where to begin. There was agreement that some of the background materials such as use of abacus, creative mathematics, easy ways of teaching Nemeth code, etc., developed by Dr. Mani in training teachers of mathematics would form an excellent base for the learning package. Following a series of deliberations by Dr. Mani, Ms. Plernchaivanich, Mr. Ramesh and myself during May and June 2003, it was decided to develop a mathematics package that would address the following major areas:

a)   Methodology of Teaching Mathematics,

b)   Use of Abacus,

c)   Use of the Nemeth Mathematical Braille code,

d)   Instructional Strategies, and

e)   Creative Mathematics

HOW THE WORK EVOLVED

Concept Building Workshop

The team wanted to find out to what extent teachers and students liked the idea of developing the mathematics package and also wanted to get a feedback on the process by developing some sample materials. A workshop was convened on July 28 - 29, 2003 in Bangkok, Thailand for two regular classroom mathematics teachers, two special teachers, two sighted students, two visually impaired students, and two blind adults who had

Participants and Project Team at the Bangkok workshop

completed their education. The two- day workshop was an opportunity to brainstorm the need and to get a feedback on the methodology proposed by the project development team. Some of the sample materials such as the tactile diagrams, paper-folding, etc., were used and the participants in the workshop expressed satisfaction with the process and the need to develop many more such materials.

Development of Instructional Materials

While instructional materials for abacus, mathematical Braille codes, etc., were available, to some extent, the project the team wanted to develop instructional strategies for teaching mathematical concepts. The teachers who interacted with the team during the initial phase of the

Project Team with teachers and visually impaired students

project indicated that adaptation techniques to suit the learning styles of visually impaired children were not clear to the teachers in special and inclusive schools and therefore, they

wished to have clear instructional material on how to teach specific mathematical concepts to children with visual impairment. Therefore, the team decided to list all the mathematical concepts found within secondary level mathematics textbooks. Frequency tables were prepared and the team arrived at a total of approximately 500 mathematical concepts covering the areas of algebra, geometry, trigonometry, statistics, etc.

Sample instructional materials were prepared by the project team and circulated to mathematics teachers and student teachers to determine the difficulty level in following such procedures in teaching mathematics to visually impaired children. The team then decided to take each concept, provide a mathematical definition for that concept along with a suggested methodology Additional materials for other areas such as the creative mathematics, developing modules for general procedures in teaching mathematics, systematic use of Nemeth codes, etc., were also developed simultaneously.

Initial Tryout of Concepts

Many individuals provided periodic feedback as the mathematical package evolved and much of that has been incorporated into various sections of this publication. Some participants helped by suggesting instructional procedures for new secondary level mathematical concepts.

With assistance from Resource for the Blind (Philippines) ON-NET organized a workshop at the Caliraya Recreation Centre, Laguna, Philippines from 18 – 22, August 2003 to gain the views of practicing teachers regarding the draft mathematics package. This workshop involved two groups; one consisting of special teachers of visually impaired children with or without mathematical background and the other consisting of teachers of mathematics who did

Demonstration of Teaching Materials

not have experience in teaching visually impaired children. In addition to instruction in teaching mathematical Braille codes, use of abacus, etc., the participants were provided with sample instructional material for about 200 concepts from secondary level mathematics and asked to critically analyze them for clarity and utility in teaching mathematics to children with visual impairment.

The feedback received by the development team was very encouraging. The mathematics teachers who did not have experience in teaching children with visual impairment felt that that the instructional package would not only be useful for special teachers but for all who teach mathematics. The special education teachers felt that the instructional materials would be of great use for non-mathematics special teachers in teaching mathematics to visually impaired children in the right manner.

Concept Development

In order to substantiate the views expressed by the teachers who attended the workshop, children with visual impairment studying at the secondary level were brought to the classroom for demonstration purposes. The teachers were asked to select concepts from the secondary level materials, prepare necessary teaching aids and teach those concepts to children with visual impairment and determine their level of comprehension of the concept introduced. The results exceeded the best expiations of the development team. The demonstration

Learning by doing

classes, which lasted for the whole day, were useful from the point of view of both the learners and teachers. Visually impaired children too expressed their enthusiasm and said they could easily understand mathematical concepts when so vividly presented in incremental steps. At the end of the session, it was the unanimous view of all that the evolving mathematical package would be useful for the following purposes:

1. To orient the mathematics teachers in regular classes to understand the techniques to be adopted in converting visual ideas of mathematics into non-visual experiences so as to help children with visual impairment understand the concepts clearly.

2. To assist the non-mathematics special teachers to understand the content in mathematics at secondary level so as to teach visually impaired children effectively.

3. To help teacher training programs to present mathematical codes, instructional methods, and mathematical devices to the trainees in an understandable manner.

4. To conduct summer programs for secondary level children with visual impairment to teach mathematical concepts and codes so that their learning of the subject is enriched.

Motivated by the positive feedback received from the special teachers, subject teachers and visually impaired children, the participants of the workshop were encouraged to work in small groups and develop instructional materials for about 50 additional concepts in secondary level mathematics which were further edited and modified by the project development team.

At this point in the projects evolution instructional materials were ready for about 250 concepts in mathematics but the project team still had to develop adaptation techniques for another 250 concepts. The work commenced at the respective work places of the project team and they exchanged information through electronic media. At every stage, the feedback from teachers was obtained.

Two workshops – one consisting of special teachers working in inclusive settings and the other consisting of mathematics teachers were conducted at the International Human Resource Development Centre for the Disabled where the ICEVI Secretariat is located and the suggestions provided by the participants were incorporated into the learning package.

Though many persons provided feedback to the materials included in the package, especially to the sections on instructional materials and creative mathematics, one person should be singled out for his generous support and excellent technical assistance in writing the material. He is Mr. T. Dharmarajan, a retired teacher of Mathematics who won the Best Teacher Award from the President of India in 1998. Though he did not teach children with visual impairment, he analytically looked at each concept from the perspective of a non-visual learner. His contribution was immense and the project development team is indebted to him for his significant support.

Planning of Master Trainers Workshop

Convinced that the approach adopted for the development of the package was sound, the ON-NET setout to organize a workshop for selected teachers and teacher educators from the Southeast Asia region to learn the entire mathematical package and get a feedback on how it could be further refined before it was released as a learning package. Keeping in mind the need to develop Master Trainers ON-NET proposed a three-week workshop using the package for selected professionals from Cambodia, Indonesia, Laos, Malaysia the Philippines, Thailand and Vietnam. Knowledge in mathematics, experience in teaching visually impaired children and a working knowledge of English were used as the criteria to select participants.

After making a thorough search for such qualified trainees, the following were selected:

1.   Ratchaneekorn Thongbai, Thailand
2.   Suladda Bupasiri, Thailand
3.   Somwang Pi Lathong, Thailand
4.   Ch'ng Hwa Lian, Malaysia
5.   Kway Eng Hock, Malaysia
6.   Ali Mushofa, Indonesia
7.   Anton Noornia, Indonesia
8.   Chan Sok Ieng, Cambodia
9.   Yim Phally, Cambodia
10.   Evelyn Bielannin Caja, Philippines
11.   Corazon Billedo, Philippines
12.   Vongnaly, Laos
13.   Hieu, Vietnam
14.   Hang, Vietnam

Feedback from Master Trainers

The ON-NET sponsored workshop was conducted by the project development team for these Master Trainers from April 22, 2004 to May 7, 2004 at the ON-NET Regional Center at Ratchasuda College, Mahidol University, Thailand. In addition to learning about the materials the trainees were also asked to provide feedback on all modules of the learning package. Peer tutoring, self-learning

Participants of the master trainers workshop

and group discussions were featured during the workshop and the responses of the trainees are summarized below:

1.   The method of teaching Nemeth Braille codes infused in them a sense of discovery and as a result, they were able to learn the codes by applying logic and certain rules and not by memorization.

2.   The self-instructional procedures prescribed for learning abacus were easy to follow but learning of abacus skills itself required more time.

3. Though specific countries had slight differences in terms of the content of mathematics at the secondary level, 90% of the concepts used under the instructional strategies section were common and therefore, the strategies could be used without difficulty.

4. In order to teach mathematics effectively, visually impaired children should be provided with Braille books in mathematics which are not available in many places.

5. The idea of paper-folding provided insights into teaching concepts in algebra and geometry in an understandable manner.

6. The instructional strategies and the creative mathematics sections should include pictorial representations to make materials more self-instructional.

The trainees also went through the descriptions of each concept listed under the instructional strategies section and edited language, content, and examples, wherever needed.

Country level activities

It was also decided at the end of the workshop that the trainees would go back to their countries and teach mathematical Braille codes and mathematical concepts to visually impaired children in the way they learned them during the workshop and bring feedback for the follow-up workshop. ON-NET supported several country level workshops where the trainees transmitted their knowledge acquired during the Master Trainers' workshop to teachers at the country level.

The work was continued on creative mathematics module and the content too was refined. At this stage the contribution of Mr. T. Dharmarajan is particularly noteworthy. He worked closely with the project team at the ICEVI Secretariat providing important feedback on the package.

Development of Mathematical Transcription Software

While the workshops and feedback from all stakeholders reassured us that the possibility of improving mathematics instructional methodology was an objective that could be attained; the lack of mathematical text materials in many parts of Southeast Asia remained a serious and unanswered challenge that would result in slowing down progress.

Although development of affordable software was not on the task list of the project development team, the challenge was there and exploration started. M/s Webel Mediatronics, Kolkata, India a software company involved in developing Braille translation software was approached by ICEVI. Dr. Mani, Secretary General, ICEVI, provided all of the

technical input needed in the development of software to prepare mathematical Braille text without difficulty.

The criteria used in developing this software were:

1. The software should enable reproduction of visual format of the mathematical text on the screen. For example, for typing triangle ABC, the shape triangle should preceed the letters ABC. The same possibility should also be available on this software.

2. The codes used at the secondary level mathematics should be grouped under different categories such as functions dealing with fractions, shapes, algebraic operations, inequalities, etc., so that the transcriber knows where to locate the inserted figure, shape, symbol, etc.

3. Conversion into Braille format should be made as simple as possible.

4. The possibility of looking at the print and Braille text simultaneously for editing purposes should be explored.

5. The software should be made Windows based and must be made compatible to work with all types of Braille embossers.

6. The software should be made available for an affordable price.

Webel Mediatronics was asked to provide the upfront development costs. ICEVI would, in turn discuss the possibility of using it for training purposes along with the mathematical package when the above conditions were fully met.

This unforeseen work continued and within a period of 6 months, the ICEVI-WEBEL effort resulted in the production of a software that facilitates production of Braille materials in mathematics using Nemeth Braille code. A series of meetings were conducted with the software engineers regarding the refinement of the package and its scope for improvement. The use of this software for mathematics textbooks in various languages will reveal the efficacy of the software, which will be modified over a period of time. Those wishing further information and updates on this *WIMATS Software (Webel-ICEVI Mathematics Transcription Software)* should direct inquires to the ICEVI Secretariat **sgicevi@vsnl.net.**

The Mathematical Package
This publication is a result of three years of research and development by a dedicated project development team co-chaired by Dr. Mani and Ms. Plernchaivanich. However this

body of work represents hundreds of contributions, large and small from teachers throughout Asia and to them we owe much thanks.

We hope you will enjoy using this publication. The material is divided into five sections as detailed below:

1. **Methodology of Teaching Mathematics**

   This section deals with the general methods used for teaching mathematics in general and teaching the subject to visually impaired children, in particular. This section also enumerates the methods of preparing mathematics text material, learning characteristics of visually impaired children, and evaluation procedures in mathematics.

2. **Use of Abacus**

   This section provides detailed self-instructional procedures to learn abacus effectively. The exercises include addition, subtraction, multiplication, division, fraction, decimals, square roots, and percentages.

3. **Use of Mathematical Braille Code**

   This section provides illustrations on how to use Nemeth Braille code for all mathematical notations at the secondary level. Each code is described in detail and accompanied by illustrations.

4. **Instructional Strategies**

   This section provides instructions on how to adapt procedures for teaching nearly 500 mathematical concepts at the secondary level. This section is also helps non-mathematics teachers to understand concepts before teaching them to visually impaired children.

5. **Creative Mathematics**

   This section deals with a whole range of creative activities such as using paper folding and the natural environment for teaching and understanding mathematical concepts.

With this background, the project development team invites you to review this new mathematical package and to start learning mathematical concepts and approaches that will enable you to teach mathematics effectively to children with visual impairment.

Lawrence F. Campbell
Project Director, Overbrook-Nippon Network on Educational Technology
and President, International Council for Edcuation of
People with Visual Impairement

August 1, 2005

Section 1

Methodology of Teaching Mathematics

APPROACHES IN
LEARNING MATHEMATICS

The possibility of the learning of mathematics by children with visual impairment is often questioned by highlighting some of the areas in mathematics which demand vision. However, many such visual ideas can be converted into non-visual experiences so as to enable them to get the required learning experience. This module describes various factors contributing to the learning of mathematics and also explains the different approaches in learning this subject.

Before discussing the different learning approaches for mathematics in the case of children with visual impairment, it will be worth mentioning the various factors which contribute to the child's success in learning mathematics. These factors can be listed as follows:

1. *Selection and teaching of suitable mathematical Braille codes.*

2. *Adaptation of the text material to the visually impaired child without changing the learning outcomes of the topics.*

3. *Teaching of mathematical devices such as the Abacus, Taylor Frame etc., to the visually impaired child for making necessary calculations.*

4. *Provision of correct mathematics text material after necessary editing of the content and format.*

5. *Preparation and use of appropriate teaching aids to supplement instruction in mathematics.*

6. *The methodology followed by the teacher in teaching mathematics has a direct bearing on the learning of the child. Provision of simulating experiences, creation of situational approaches, etc., can enrich learning in mathematics.*

7. *In an integrated setting, the support of the regular classroom teacher is also important in teaching mathematics to the visually impaired child. The complementary roles of the resource and regular teachers may ultimately benefit the visually impaired child in attaining appropriate learning experience.*

Mastery over Mathematical Braille Code

The Nemeth Code and the RNIB English Braille Code are widely used for teaching mathematics to children with visual impairment. In addition to these codes, some countries have also adopted their own codes. This package is using the Nemeth code. Whatever may be the code, the real challenge lies in teaching it to the child effectively. To make the child learn the mathematical Braille codes effectively, certain teaching points have to be considered.

a. *Teaching of the code should start from the primary level itself. As soon as the child learns writing, strenuous practice in writing mathematical codes is to be given. Since experiences have shown that visually impaired children show resentment to learn mathematical codes when they are introduced at a later stage, say, directly at the secondary level, the teaching of these codes should take place in a phased manner from the very beginning of the child's schooling.*

b. *It is not necessary to teach all mathematical codes at a time. Suppose 15 codes are to be taught at the III Standard, these can be taught as and when they are required in the lesson. This way of introducing the appropriate code at the appropriate time makes the learning spread out throughout the year and it will be more meaningful to the child.*

c. *When the child is through with the writing of the code after the writing practice, a small passage can be prepared in mathematics and the child be asked to read it. This is treated most essential because the child should know how to discriminate the mathematical Braille code from the literary code while reading. This practice should be continued until the child gets mastery over the usage of that code.*

ICEVI

Adaptation of the Mathematics Text Material

In the process of providing substantial learning experience to children with visual impairment, it is advisable to keep the expected outcomes on par with sighted children and adapt/ substitute learning experience to derive maximum understanding of the concept. Take for example, the concept of "Rows and Columns", an effective teacher will be able to create a lot of situations to explain this concept to the child. In fact, the cells of the Braille slate can be used to explain this idea; the Geo-board can be used, the seating positions of the children in the class itself can be used to explain this, tactile graph sheets can be used and so on. Even though it is primarily true that certain concepts are seen and understood, the fact remains that most of the concepts could be modified to suit the needs of the visually impaired child.

Development of Mental Arithmetic

The mental ability of doing mathematical calculation is the result of concentration and mastery over the basic mathematical operations. Since no single device would be exclusively useful for visually impaired children for their calculations in mathematics, it is quite reasonable to look into the task of developing mental arithmetic abilities in them. Like all other activities, this too needs systematic instruction, practice and application. In visually impaired children, this exercise could start with the learning of abacus. The calculations in the abacus require the mastery of addition and multiplication facts (for example, multiplication is the easy process of adding many times) and the speedy process in the abacus contributes to the mental abilities in calculations. Once the child is very proficient with the operations in the abacus ranging from addition, subtraction, multiplication, division (especially long division involving many digits) upto the process of calculating the square root, percentage, etc., he/she can be trained to use the short-cut techniques in computing values. For example, 458×208 can be broken into various steps such as 400×208, 50×208, 8×208 and even to further steps depending on the ability of the child to store the calculated values in brain before making the sum total of the entire calculation. Training in remembering a set of numbers over a period of time, games for calculations, etc., can be performed by the student and the teacher for getting sufficient practice. Prolonged training and practice in performing mental calculations help children to acquire the mathematical mind which is very essential for problem solving, analysis of information, scientific approach in performing the day-to-day activities, etc.

After each exercise in improving the mental calculations, it is necessary for the child to verify his answer either with the use of an abacus or with the Taylor Frame. The process of calculation in the mathematical device may help him to find where he makes mistakes in the process of mental calculations. Since the early years are most conducive for the development of such skills, the child should be oriented to this type of learning from the primary level itself.

Methods of Preparing and Using a Mathematics Braille Text Material

Most of the teaching of mathematics to sighted children is done through blackboard work supplemented by oral instruction. The oral instruction makes sense to children with visual impairment only when the matter is presented either through visual or tactile information. Therefore, the need for mathematical text is inevitable. It is however noted that provision of mathematical text material will be of less value when the child is not proficient with the mathematical Braille codes. Therefore, teaching of mathematics and the using of mathematics text material should always go along with the teaching of the Braille codes.

Unlike the tactile material used for other subjects, mathematics text material involves ideas such as literary Braille code, numerals, symbols, formulae, diagrams etc. Orientation and practice of transcribing such codes into the Braille form would be necessary for every teacher of visually impaired children.

Material for visually impaired children is prepared bearing in mind the suitable principles of maximisation of the duplicated material, the modification of format and content for necessary adaptation, the substitution of ideas/lessons for optimum learning experience of the child and rare omissions under unavoidable circumstances. Mathematics being an abstract subject, which involves the appearance of concrete, pictorial and abstract concepts, all the four principles should be used in preparing text material.

The second important aspect is to incorporate necessary diagrams in the lessons. Provision of such diagrams in the text itself enhances the understanding of the child. It should be noted that the diagram is not overloaded with information.

To supplement the use of mathematics text material, a small guidebook in Braille consisting of the model problems, certain diagrammatic illustrations, etc. can be provided to the

child. This helps the child to revise especially during the examination time. Though a complete mathematics lesson cannot be recorded into the audio form, certain steps such as the formulae, methods of constructing a diagram in the case of geometry, etc., can be recorded into a cassette. All possible alternatives have to be explored for making the teaching-learning process in mathematics more purposeful.

In case the entire mathematics text material is not available in the Braille form, the teacher can prepare at least the necessary chapter or portion of the text which is to be taught in the particular class period. Provision of such concrete material is most imperative as it acts as a guide for reference in remembering various steps of a problem, procedures of arriving at results, etc. Literary text material can be presented in other ways, viz., the live reading and recording services, but mathematics text material needs to be presented in the tactile form. The teacher should see that the visually impaired child has the right reading material in mathematics class so that he can develop his mathematical skills in a gradual and natural manner.

Mathematics is the subject which is to be practised continuously if one wants to have mastery over it. Unlike other subjects, this demands a lot of attentiveness, reasoning ability, skill in problem solving, ability in drawing conclusion, etc. Once mathematics was considered to be an impossible educational experience to visually impaired children, but with the development of various devices and effective teaching methodologies, they can also learn mathematics effectively and it is up to the teacher and the student to make full use of the available resources.

IMPROVISATION OF
TEACHING AIDS IN MATHEMATICS

Teaching aids are of many types. There are sophisticated aids, teacher-made aids and even some other real objects and materials of the environment which can be used as teaching aids. Whatever might be the aid, the usability of it depends on the nature and need of the learner, the emphasis the teaching aid lays on learning the concept and the ability of the instructor to make use of the teaching aid to the maximum extent. This module highlights the importance of three-dimensional aids and the need and nature of improvisation of aids in the education of children with visual impairment, particularly in the area of mathematics.

Tactile Tolerance

Since vision plays a predominant role in the assimilation of ideas by observation, certain aids which are commonly available for sighted children have to be adapted to suit the needs of visually impaired children. The concept of tactile attraction is to be emphasised in preparing teaching aids. Take for example, the Venn diagram explaining the concept of "intersection". In the diagram, the two sets A and B and the intersection $A \cap B$ are presented in various colours. The same can be adapted to the use of visually impaired children by pasting sand papers of different coarseness to represent different areas. If this diagram is to be used by the student of higher standard who has acquired the ability to discriminate even the finest textures, the selection of the level of roughness is immaterial. But on the other hand, for the child who is having limited skills in discriminating textures, various parts of teaching aids should have distinct differences in textures. Therefore, the texture selection also depends on the mental maturity of the visually impaired child who uses it.

Need for Improvisation

Improvisation is important for various reasons. Firstly, procuring sophisticated teaching aids demands resources. Secondly, the education of visually impaired children warrants more varieties and, as a result, a maximum number of teaching aids would be required. The teacher should know the techniques of using available resources which can provide maximum simulating experiences to the visually impaired child.

Wealth from Waste

In a normal school environment, many things which are usually treated as waste can effectively be used as teaching aids for visually impaired children because what they feel is more important than how the teaching aids look like.

For example, a waste chalk box can be used to explain the concept of a cubical; a rubber ball can be used for explaining the concept of a globe; different plastic balls can be used to explain the concept of volume; the chalk bits and stones can be used for counting; the Braille book itself can be used to explain the concept of rectangle/square; the waste cardboards can be used by the teacher to prepare cut-outs of triangles and various geometrical figures and so on. Making wealth from waste depends on the creativity of the teacher.

Adaptation of Aids

As indicated in previous paragraphs, tactual attraction at every stage is more important and the teacher should verify that the child does not encounter any difficulty in understanding the concept. Geometrical devices can also be adapted to the needs of visually impaired children. Take for example, the protractor. The student is expected to measure the angles with the use of this equipment. In the normal protractor made out of plastic, big holes can be made for every ten degrees and small holes for every five degrees without breaking it. A small needle can be used to grave such tiny holes. It is however noted that minute information such as the dots for every unit cannot be made. If the teacher wants the graved points to have an embossed feeling, a drop of glue can be put in such holes which will give embossed feeling very distinctively. Similarly, many normal teaching aids can be improvised for the needs of visually impaired children. The main thrust of improvisation is to make adaptations more effective, to prepare many teaching aids with less cost and to provide varied teaching aids.

Teachers of visually impaired children with the guidance of experts in the subject (if needed) can adapt such teaching aids for the use of the visually impaired children without incurring much cost.

Three-dimensional Aids

There is no doubt that three-dimensional aids would give concrete experiences to the visually impaired child in understanding a specific concept. On the other hand, certain models may not give the real experience to the child when he/she has not conceptualised it as a whole. For example, a small model of a multi-storeyed building or a big mountain may not provide the real experience to the child but the child can get an idea about those. This knowledge acquisition depends upon the age of onset of blindness too. Naturally, blind children who have seen the objects before becoming blind can comprehend them in a better way.
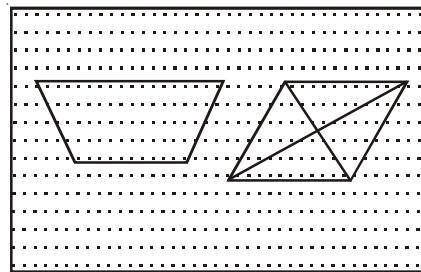
In mathematics, most of the teaching aids can be presented tactually because they are aiming at the development of certain concepts. Area, volume, height, weight, elevation, scale value, etc., are some concepts which can be effectively explained through three-dimensional teaching aids. There are three-dimensional aids available from the market to teach the above ideas. But the teacher should assess the usability before procuring such aids. The following principles are very important for the selection of three-dimensional teaching aids:

i.   *The three-dimensional teaching aid should be handy. It should not be too big to explore or too small to understand the minute differences.*

ii.   *It should be strong and sturdy so as to withstand the manipulation of the visually impaired child.*

iii. *As far as possible, sharp edges should be avoided in three-dimensional aids for visually impaired children. Sharp edges may be made blunt to avoid injuries to the Braille reading fingers.*

iv.   *If the teaching aids are of collapsible type, understanding will be better. For example concepts like hemisphere, diameter, circumference, radius, etc., can also be explained when the globe is of collapsible type.*

The main purpose of the three dimensional aids is to make the children understand the two-dimensional interpretations of them at a later stage. The teacher should check at every stage that the children acquire this skill effectively.

Geo-Board

The geo-board is a multi-purpose board. This can be used for showing geometrical figures and graphs. It is a peg board, square or rectangular in shape with nails at equal distance, both lengthwise and width wise. The distance between the nails can be determined according to the levels of the students. For example, the distance can be brought down even to one centimeter in the case of children of higher classes, whereas it should be at least one inch in the case of primary school children. This geo-board is a wonderful companion to the teacher of visually impaired children, especially in the teaching of mathematical concepts. Rubber bands can be used to show various shapes. When the distance between the nails is smaller, even circles can be shown. A Magnetic Board with magnetic strips and magnetic pieces pasted with braille symbols may also be useful for presenting learning materials in a spatial format.

Construction

Construction activities for visually impaired children are of two types. Firstly, making three-dimensional objects out of small pieces and secondly, construction of a two-dimensional tactile figure based on the given measurements. In the first case, small blocks can be used to construct three-dimensional figures. Cut-outs can be prepared and used for explaining certain concepts. For example, the Pythagorean theorem can be explained very effectively with the use of cut-outs and the child himself can be oriented to measure the area of the squares on the base and the opposite side and compare their sum total with the area of the square of the hypotenuse. This develops the discovery skill of the visually impaired child. The circumference of a circle can be measured using a thread and the volume of the globe can be measured by filling it with water and so on. There are many

activities which can be made more meaningful for the visually impaired child by a creative teacher.

The second type of construction is the actual drawing of the diagram by the child. Even though this is complicated to some extent, there is no doubt that the visually impaired child can understand the procedures and draw figures if necessary equipment are provided. Suppose the child is expected to draw a straight line, a tactile scale can be used and the child can draw the line with the use of wax or crayon pencils. The wax can be felt later to trace the line. A wax pencil instead of a normal pencil can be used in a normal compass to draw a circle. It is very essential that the child receives adequate training in these areas before he/she uses these equipment independently.

Relief papers are also used in schools for drawing diagrams in mathematics. The relief papers which provide upward embossed impressions straightaway assist visually impaired children as well as the teacher to deal with geometry effectively. At each and every stage of the construction of the diagram, the child will be able to assimilate ideas. When relief papers are not available, the tracing wheel can be used to draw the embossed diagrams. Since this type provides downward impression, the entire diagram has to be mirrored while drawing.

The above principles highlight the importance of teaching aids which can make the learning of mathematics more interesting to the visually impaired child. Even with all available resources, a teacher who is less creative may not derive maximum benefit for visually impaired children from the teaching aids. Therefore, it is vital that the teacher tries to be more creative so that he can bring utmost variety to the education of the child.

MATHEMATICS IN INTEGRATED CLASSROOMS

The need for learning mathematics differs to some extent between residential schools and integrated education programmes. In residential setting, some specific areas in mathematics which are considered to be complicated may be omitted by the teacher while teaching. This mass omission may not bring discrepancy among the students in learning that subject since the omission is uniform for all. But the integrated setting is entirely different. The complete content in mathematics is taught to sighted students. A visually impaired child in the integrated setting may be in a disadvantageous position if he is not taught the necessary content. Hence, whether or not visually impaired children like mathematics, they have to learn it if they study in the integrated setting where mathematics is one of the subjects of study. This write-up provides guidelines on how mathematics learning can be made effective in integrated programmes with a sound collaboration between resource and regular teachers.

Learning Sequence

A good learner of mathematics will be able to 'organise' ideas. When the organisation is good, the learner will be able to put the ideas in a sequence. Good 'sequence' will be very useful for easier arrival of results. When correct results are arrived at, the 'interpretation' will be good. Though the expected outcome is the same for both visually impaired children and sighted children, the means are different. In the regular classroom, most of the teaching of mathematics is done through blackboard work supplemented by oral instruction. Sighted children grasp the idea of "organising" and "sequencing" mostly by the manner in which the matter is presented on the blackboard. Due to the limitation caused by visual impairment, the child misses this very important information in learning. Thus, visually

impairment children face difficulties in the rest of the processes, viz., the arrival of results and interpretation. Every mathematics teacher of visually impaired children should bear this in mind while planning strategies in teaching mathematics.

Common Difficulties in Learning Mathematics

The pace of learning mathematics by visually impaired children is an important issue. Their pace is comparatively 'slower' than that of sighted children. This is due to the visually impaired child's limitations in organising ideas, methods and devices used for solving the mathematical problems. The following remedial measures can be suggested to provide equal educational experience to the child. If there are 20 problems in the exercise of a chapter and if the teacher expects sighted and visually impaired children to solve all the problems within the class period, the visually impaired children will be experiencing difficulties in doing so. For example, 20 problems are listed in the textbook as exercise for explaining the ideas of 'simple interest' and the methods of using the formula for solving given problems. Doing this full exercise is practically not possible for visually impaired children. Here learning the 'concept of simple interest and its practical uses' is more important than the 'number' of problems. There are individual differences between any children in learning these concepts. Some children understand the concept just by solving two problems. Some understand after five problems and some may find it difficult to understand even after many trials. We see all these among sighted children. This pattern can also be seen among visually impaired children. If the child is able to learn and use the idea just with few examples, it is not necessary to tax him/her for doing the entire exercise. The teacher can select the problems of the exercises at random and give practice to visually impaired children. Due to the slower pace of doing mathematical calculations, the approach for visually impaired children might be "Do as many as possible for understanding the concept. As long as you are able to use your learning to tackle new problems, it is alright." Just by saying this the child cannot be expected to have learnt the ideas. Sufficient drill and evaluation must become part and parcel of the learning.

Collaborative Instructional Strategy

In teaching mathematics, the following procedures are suggested for an integrated education setting. This holds good for residential school too as the suggestions are made from the methodology point of view. The child who is admitted to the integrated programme not

later than standard VI could be taught mathematics in spite of the child's deficiency if any in mathematics in his previous classes. Here are some guidelines for resource teachers:

i) Teach abacus or Taylor frame techniques to the visually impaired child. Sometimes, the child may not develop an interest in learning that skill. Unless he/she is poor in motor skills and finger manipulation he/she is not excused from learning the suitable mathematical device.

ii) Have consultations with the regular teacher and know the content area planned by him/her for the regular class for that week. Such need will not arise in a residential set up.

iii) Go through the content. If you have difficulty in understanding the basic concepts of the content, please do not hesitate to ask the regular teacher. Keep in mind that he/she is the subject specialist. In a residential set-up, the decision is made by the mathematics teacher himself/herself.

iv) Prepare the text material. If the mathematics text is already available, that can be used. If not, decide which part of the lesson is to be given due importance. In fact, it is vital to prepare the derivations and illustrations given in the particular chapter in Braille format.

v) For one or two problems of the exercise, prepare model illustrations by giving various steps used for solving the problems. This assistance may be obtained from the regular teachers in the case of non-mathematics resource teachers in the integrated setting.

vi) Diagrams, if necessary, can be prepared to supplement the instructions given in the text material. Aids can also be prepared if they are necessary.

vii) As a 'before classroom instruction', let the child be oriented to the text material. Please do not take it for granted that the child could do it by himself. Frequent evaluation is a must, especially in the area of mathematics. Let the child also go through the model illustration. Please note that this is not teaching. This is just an orientation on how to use the material in a regular class in a better way.

viii) Let the child go to the regular class for his lesson. Please do not send the child to the regular class without reading materials. Now the lessons taught by the regular teachers

in the class would be meaningful to the visually impaired child. When sighted children solve problems of the exercises, let him solve as many problems as possible.

ix) In doing so, the child will be able to ask the regular teacher doubts, if any, regarding the content of that lesson. If he needs any assistance regarding the format and the presentation of reading materials, he could consult the resource teacher too.

x) The child should be asked to write the steps used for solving the problems. *The procedure should be written by the child*. One or two problems solved in the class can be transcribed into print for the mathematics teacher. Going through this format will help the teacher to understand the difficulties of the child in the presentation and interpretation of the results. It will also help the teacher in understanding the difficulties of the child in learning the content. This is the easiest process of diagnosing the difficulties of visually impaired children in learning mathematics.

xi) As a follow-up, teachers can conduct some tests for the child. This extra drill is necessary for the visually impaired child. These types of tests may help the child to undertake his mathematics examinations with confidence.

xii) The practice should not stop only with writing. The teachers can ask the visually impaired child to explain the procedure orally. This would enrich the understanding of the child in that content area.

When the above-said procedure is followed without any discontinuity at least for one full year, the child will be able to show significant and substantial development in his learning of mathematics. Mathematics learning is not difficult, but it is a long process which makes the child and the teacher feel it difficult. This is not achieved overnight; this needs continuous effort. "Mathematics is difficult for visually impaired children" - this statement is absolutely subjective. Some children like mathematics, some do not. Some teachers are interested in teaching mathematics and some others are not. But as a teacher of visually impaired children, one should have the basic faith and confidence in teaching mathematics.

In short, with proper material, good coordination and with a thorough follow-up of the learning, mathematics can be made easier for visually impaired children.

Section 2

Use of Abacus

ABACUS - GENERAL CONCEPTS

a) **Abacus**

Abacus is a device used by visually impaired children for doing basic mathematical calculations. Abacus is rectangular in shape. Abacuses with varied columns are used in different countries. This instructional material is written specifically for the abacus with 15 columns. The common operations for this abacus are same with those of the abacuses with fewer columns, but the number of columns matters especially when fraction problems are solved.

A bar is separating the abacus horizontally cutting across all the fifteen columns, leaving $\frac{2}{3}$ rd of the area below and $\frac{1}{3}$ rd of the area above.

The lower portion is known as lower abacus and the upper portion is known as upper abacus.

Each column in the lower abacus has four beads, each bead assumes the value 1 (one). Each column in the upper abacus has one bead and assumes the value 5 (five).

In operation, the extreme right column is treated as the units column, the immediate left to the units column is the tens column and so on.

b) **Setting**

The process of moving a bead of the lower abacus or the upper abacus towards the separation bar is called **'setting'**.

c) Clearing

The process of moving a bead away from the separation bar either towards the top of the upper abacus or the bottom of the lower abacus is called **'clearing'**.

d) Example for complement of a number with respect to the given number

For example, take the number 6. The complement of this number with respect to number 10 is 4. Hence complement is the value which 'Completes' (that is, the 'remaining' value with respect to the number which completes).

The complement of the number 6 with respect to the number 100 is 94; the complement of number 6 with respect to 1000 is 994 and so on.

EXERCISE

What are the complements of:-

i) 7 with respect to 1000

ii) 8 with respect to 10

iii) 15 with respect to 100

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e) Dots in the Separation Bar

The dots in the separation bar are helpful for locating the place values of numbers in abacus.

These dots determine the locations such as whole number, numerator, etc., when fractions are presented in the abacus. These dots may not be present in some abacuses which have 13 columns. In such cases, imaginary dots have to be assumed for placing the numerator, denominator and whole number in the abacus. This does not envisage any change in the procedures in operation.

After learning the modules on abacus, the learner is expected to demonstrate the following skills :

1. *Ability to formulate problems for addition of one digit, two digits and multiple digits and adding them in abacus.*

2. *Ability to formulate and solve problems in subtraction.*

3. *Ability to formulate and solve problems in multiplication.*

4. *Ability to formulate and solve problems in multiplication of numbers involving zero. For example 609 x 5, 210 x 98, etc.*

5. *Ability to formulate and solve problems in division.*

6. *Ability to formulate and solve problems in long division, i.e., using four digit numbers as dividends.*

7. *Demonstrating the placement of whole number, numerator and denominator in addition and subtraction of fractions.*

8. *Ability to formulate and solve at least 10 problems in fraction-addition.*

9. *Ability to formulate and solve at least 10 problems in fraction-subtraction.*

10. *Ability to formulate and solve at least 10 problems in fraction-multiplication.*

11. *Ability to formulate and solve at least 10 problems in fraction-division.*

12. *Ability to demonstrate decimal operations-placement of decimal points in the abacus.*

13. *Ability to find percentage values for 10 fractions.*

14. *Ability to find the square-roots of at least 10 whole numbers and decimal numbers.*

15. *Demonstrating the skill in using two abacuses simultaneously for solving complex mathematical problems.*

PRE-REQUISITE SKILLS FOR THE EFFICIENT LEARNING OF ABACUS

Abacus should be introduced as early as possible in schools. However, those children who did not have an opportunity to learn the skills at the primary level can also learn those at a later stage. For such children, understanding as well as mastery in certain mathematical concepts would enhance the effective learning of operations through abacus. These pre-requisite skills are only suggestive (except 1-5) and not prescriptive. They are as follows :

1. Demonstrating the correct finger movements in using abacus.

2. Demonstrating correct hand positions in using abacus.

3. Explaining the counting procedures in abacus.

4. Explaining the concept of **Clearing** and **Adding** in abacus.

5. Understanding the concept of **Complement** of certain number with respect to another number.

6. Memory of Multiplication tables for numbers 1 to 20, preferably for higher classes.

7. Memory of the **squares of the numbers** 1 to 25 (minimum), preferably for higher classes.

8. Memory of the **square roots** (perfect) of squares from 1 to 1000, preferably for higher classes.

9. Understanding the relation between fraction division and fraction multiplication.

10. Understanding the concepts of **Least Common Multiple** and **Highest Common Factor**.

ABACUS - ADDITION

I.   GENERAL RULES :

a)   Finger position is very important in operating the abacus. The fingers should not 'butterfly' over it. The thumb should operate ONLY the lower abacus and the index finger should operate the upper abacus. On any account, the positions should not be changed.

b)   Take this example : 48 + 67

Here, the number 48 should be set in the extreme right of the abacus - the number 8 in the units column and 4 in the tens column.

If you need the number 67 also in the abacus for remembering, set it in the left extreme of the abacus. There is no hard and fast rule for setting this number in the particular column in the left side of the abacus. This is just for REFERENCE.

c)   One of the important aspects to remember in abacus is the position of the two hands on that. The left index finger and the right index finger should always be placed on the consecutive columns for avoiding confusion. The placement of the hands should be emphasised very much while teaching the visually disabled child. Otherwise, children will miss track of the problem.

d)   In the addition of numbers, the digit in the units column of the first number should be added with the corresponding digit of the second number. Similarly the corresponding digits of the numbers must be added. On any account, the units column of the first number and the tens column of the second number should not be added. This procedure is just like the addition techniques followed by sighted children. However a small difference is noted. In normal addition,

we start with units column and proceed to the higher ones. In the abacus, we start with the highest digit column. In the above example, we have to add the tens column of the two numbers first (i.e., 4 and 6) and the units column (i.e. 8 and 7) next. This is for easy calculation through abacus.

e. While you move your left hand to the immediate left of the particular column for setting a number, your right hand should automatically crawl to the left, following the left hand.

f. Before taking the new problem, verify that all the beads of the abacus are cleared.

g. The teaching of subtraction may also be carried out along with abacus addition.

II. ADDITION OF SINGLE DIGIT NUMBERS :

Following are the important steps used in adding two numbers in abacus :

A. *Example : 4+5*

a) First, set the number 4 in the units column. That is, move all 4 beads in the units column towards the separation bar.

b) We have to add 5 in the units column. Since there is no bead to add in the lower abacus, set the only bead of the upper abacus (with your right index finger) which takes the value five.

c) Count the number in the units column. The answer is 9.

B. *Example : 4+3*

a) Set the number 4 in the units column. (Please see that the hand positions are correct).

b) We have to add 3 in the units column.

c) Since there is no bead available in the lower abacus, go to the upper abacus. The only bead which has the value 5 can be moved towards the separation bar.

d) What have you done in the previous step? Instead of adding 3, you have added 5. Therefore, the excess 2 must be subtracted. Clear two beads of the units column (in the lower abacus). Now the remaining value is 7, which is the required answer.

III. ADDITION OF TWO DIGIT NUMBERS :

In addition, the higher value digits are always added first. The units column digits will come at last.

Example : 37+36

a) Set the number 37 in the extreme right of the abacus. In setting and clearing beads, we must be very careful in moving hands. That is, set the number 3 of the 37 in the tens column with the right hand and set the number 7 of the 37 in the units column with the same hand after setting number 3. Left hand rests on the 3 in the tens column while the right hand is on the units column.

b) We are going to add 36 with 37. That is, we have to add 3 in the tens column and 6 in the units column.

c) Since there is no 3 to add in tens column in the lower abacus, we can add 5.

d) Instead of adding 3, we have added 5.

e) Subtract the excess 2 in the tens column.

f) Next, we have to add 6 in the units column. There is no 6 to add in the units column. Only two beads are remaining in the lower abacus.

g) Therefore, we shall go to the tens column. That is, set a bead in the tens column. It means you have added 10 instead of 6. Therefore, subtract the complement 4 from the units column.

h) Since there is no 4 to clear in the lower abacus in the units column, clear 5 of the upper abacus and add 1 bead in the units column in the lower abacus.

i) Count the number in the abacus. The answer is 73.

IV. ADDITION OF MULTIPLE DIGIT NUMBERS :

Example : 987 + 444

a) Set the number 987 in the extreme right of the abacus.

b) We are going to add 444 with 987. Start with higher value digits. That is, we have to add 4 in the hundreds column, 4 in the tens column and 4 in the units column.

c) Since there is no bead to add in the hundreds column, we cannot add directly.

d) We shall add one bead in the column which is the immediate left of number 9. That is, number 1 is set in the thousands column. This means that we have added 10 hundreds. That is, instead of adding 4 hundreds we have added 10 hundreds. Therefore, we have to subtract 6 hundreds from the hundreds columns.

e) Clear 6 from 9 in the hundreds column.

f) This process can continue with the next number (i.e., tens column numbers). As in the previous case, it is not possible to add 4 with 8 directly in the tens column.

g) Add one more bead in the hundreds column and clear 6 beads in the tens column. That is, clear 6 from 8 in the tens column.

h) In the same way, to add the number 4 in the units column, add one bead in the tens column and clear 6 from 7 in the units column. The answer is 1431.

EXERCISE

1.	434	+ 528	6.	60001	+ 9979
2.	308	+ 594	7.	8940	+ 1239
3.	1234	+ 49	8.	9009	+ 999
4.	7894	+ 534	9.	100020	+ 80728
5.	59819	+ 9888	10.	150711	+ 4304

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ABACUS – SUBTRACTION

The rules followed in abacus addition are applicable in subtraction too. The highest digit in the smaller number should be subtracted from the corresponding digit of the higher number and so on. The procedures are explained in the following problems.

A. *Example : 43 - 32*

    a)   Set the number 4 of the 43 in the tens column with right hand and set 3 of the 43 in the units column by moving hands to the right.

    b)   Like addition, subtraction is also done from left to right. In this example, we have to subtract 3 from 4 in the tens column and 2 from 3 in the units column.

    c)   Place your right hand on the number 4 of the 43 and clear 3 beads.

    d)   Move both hands to the right, clear 2 beads in the units column.

    e)   Count the number in the abacus, the answer is 11. This is DIRECT subtraction.

B. *Example : 82 - 19*

    a)   Set the number 8 of the 82 in the tens column.

    b)   Set the number 2 of the 82 in the units column. Now your right and left hands should be on the units and tens column respectively.

    c)   In this example, we have to clear 1 in the tens column and 9 in the units column.

    d)   Clear one bead in the tens column and move to the right to clear 9 in the units column. Since the units column has only a value 2, it will not be possible to clear 9 in that column.

e) With the help of your left hand, clear one bead in the tens column. That is, you have cleared 10 instead of 9.

f) To compensate this excess in clearing, add one bead in the units column (i.e., add one bead with the already available two beads). The answer is 63.

C. *Example : 378 - 179*

a) Set the number 378 in the extreme right. That is, set the number 3 of the 378 in the hundreds column, 7 in the tens column and 8 in the units column.

b) In the given example, we have to subtract 1 in the hundreds column, 7 in the tens column and 9 in the units column.

c) Clear one bead in the hundreds column. You have 2 beads left now.

d) Clear 7 in the tens column (left hand follows the right and rests on 2 in the hundreds column). You have no beads left now in the tens column.

e) Move both hands to the right to clear 9 in the units column. It is not possible because you have only 8 in the units column.

f) Move your hands to clear one bead in the tens column. Incidentally, there is no bead left in the tens column. This is typical, isn't it? Now move your hands to the left. Clear one bead in the hundreds column. This means you have cleared 100 instead of clearing 9.

g) What is the complement number of 9 with respect to 100? It is 91. This number must be added to compensate the excess. That is, add 9 beads in the tens column and one bead in the units column. In the tens column, there is no bead. So your addition results as 0+9=9. In the units column, you have the value 8 and to this value, 1 bead is added making it 9. Thus you get the answer as 199.

EXERCISE

1.	984	–	234	6.	8324	–	524
2.	842	–	599	7.	105711	–	4305
3.	1024	–	55	8.	232824	–	99821
4.	10889	–	7024	9.	172844	–	81234
5.	5904	–	4801	10.	334781	–	70024

ABACUS - MULTIPLICATION

I.   GENERAL RULES :

a)   The finger and hand positions are to be followed as in the case of addition and subtraction.

b)   In abacus multiplication, if you get the multiplied values as 1, 2, 3, 4, 5, 6, 7, 8 and 9 after multiplication of two digits, they should be treated as 01, 02, 03, 04, 05, 06, 07, 08 and 09 respectively. This is very essential for having correct column placement in the abacus. Example : 3 x 2 = 6. This should be treated as 06.

c)   In 67 x 78, 67 is the number by which the multiplication is done and is called the **multiplier.** 78 the number which is multiplied, is the **multiplicand**.

d)   The multiplier is always set in the extreme left.

e)   For setting the multiplicand in the right side of the abacus, count the number of digits of the multiplier; add that number with the number of digits of the multiplicand and ADD ONE MORE COLUMN for abacus multiplication. For example, in 45 x 225 the multiplier is a two digit number, the multiplicand is a 3 digit number and they have a total of five digits put together; add one more column to this. It will become six. The multiplicand should be set from the last but sixth column in the right side of the abacus.

f)   Multiplication is a rapid form of addition.

g)   The multiplier and the multiplicand together are called FACTORS.

h)   The highest digit of the multiplier and the lowest digit of the multiplicand should be multiplied first.

i)   When multiplication of one digit with the multiplier is over, the completed digit of the multiplicand should be immediately cleared before taking up the next digit of the multiplicand for multiplication. The multiplied value will always appear in the right side of the multiplicand.

j)   When the placement of the multiplicand is correct, the digits of the product will appear in their respective places.

II.   MULTIPLICATION

A)   *Multiplication of Single Digit Numbers :*

Example : 4 x 7

a)   The multiplier 4 is placed in the left extreme of the abacus.

b)   Move both hands to the extreme right to find out where should we place the multiplicand.

c)   For that, count the number of digits in the multiplier and multiplicand and add one column or digit for the abacus.

d)   In this example, there is only one digit in the multiplier and one digit in the multiplicand and add one for abacus. The total is 3 (places).

e)   From the extreme right, count 3 places: - first column-units, second column - tens and the third - hundreds. Therefore, we have to place the multiplicand 7 in the hundreds column with right hand.

f)   Multiply the multiplier and the multiplicand. That is, 4 x 7 = 28. Set the number 2 of the 28 in the tens column and the number 8 of the 28 in the units column.

g)   Clear the multiplicand and the multiplier. The number left in the extreme right of the abacus is the "PRODUCT". The answer is 28.

EXERCISE

1) 8 × 9

2) 5 × 8

3) 4 × 9

4) 6 × 5

5) 2 × 8

6) 3 × 5

7) 8 × 4

8) 9 × 7

9) 4 × 3

10) 7 × 6

ABACUS - MULTIPLICATION OF TWO DIGIT NUMBERS

Example : 38 × 29

a)   Set the multiplier 38 in the extreme left.

b)   Count the digits of the two numbers (multiplier and multiplicand) and add one for the abacus. Totally, we have 5 digits. Therefore, set the multiplicand 29 in the last but 5th column in the right side of the abacus.

c)   Keep the right hand on 9 of the 29, the left hand on 3 of the 38 and multiply. 3 × 9 = 27.

d)   Set the number 27 in the immediate right of the multiplicand. Now multiply 9 of the multiplicand and 8 of the multiplier. The value is 9 × 8 = 72. Please note that this number should be added with where you have left in the multiplication of the previous digit 3 of the multiplicand.

e)   Set the number 7 of the 72 in the tens column. You have already number 7 in the tens column. Therefore, there is no 7 to add in that tens column. Therefore, set one more bead in the hundreds column and subtract 3 from the tens column. Set the number 2 of 72 in the units column. Since the first step is over, clear the number 9 of the 29.

f)   Multiply the number 2 of the multiplicand with the multiplier 38, i.e., 2 × 3 is 6. Treat this as 06. Skip one column for zero and set the number 6 in the hundreds column.

g)   Multiply 2 × 8 = 16. Set the number 1 in the hundreds column. But there is no bead to add in the hundreds column. So, set one bead in the thousands column and clear nine beads in the hundreds column. Now we have to add 6 in the tens column. We have no place there. So, add one bead in the hundreds column and clear 4 beads in the tens column.

h)   Clear the multiplicand and the multiplier. Now you have 1 in the thousands column, 1 in the hundreds column, 0 in the tens column and 2 in the units column. The answer is 1102.

EXERCISE

1)   23 × 45

2)   33 × 57

3)   21 × 78

4)   47 × 39

5)   73 × 81

6)   49 × 51

7)   38 × 58

8)   49 × 76

9)   98 × 71

10) 41 × 38

ABACUS - MULTIPLICATION INVOLVING ZERO

Example : 405 x 307

a)   Set the multiplier 405 in the extreme left.

b)   For setting the multiplicand, count the digits of multiplier and multiplicand. Add one digit for abacus. Totally we have 7 digits. Therefore, set multiplicand 307 in the right side, the number 3 in the last but 7th column and so on.

c)   Now, we are ready to multiply. Multiply the number 7 of the 307 and the number 4 of the 405, i.e., 7 $\times$ 4 = 28. Set the product 28 in the immediate right of 7 in the right side of the abacus. Keep your right hand on the number 8 of the product.

d)   Next, 7 $\times$ 0 is 0. Therefore, skip the column where your right hand rests and move both hands to the right and keep them on the hundreds and tens column respectively.

e)   Multiply 5 $\times$ 7. That is 35. Place the 3 in the tens column and the number 5 in the units column. Clear the number 7 of the multiplicand.

f)   Since the multiplication has a zero in the middle, just leave it. Let us multiply the number 3 of multiplicand with the number 4 of 405. 3 $\times$ 4 is 12. Set the number 12 on the immediate right of the multiplicand with right hand. Keep your right hand on the 2 of the new number 12.

g)   Next, 3 $\times$ 0 = 0. Therefore, skip that column and move both hands to right. Keep them on the next consecutive columns. Multiply 3 $\times$ 5 i.e., 15. Set the number 1 in the thousands column and 5 in the hundreds column.

h)   Clear the 3 of the multiplicand and the multiplier 405. The answer is 124335.

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EXERCISE

1)   304   × 408

2)   708   × 601

3)   741   × 509

4)   909   × 869

5)   4321 × 109

6)   1009 × 709

7)   4089 × 2089

8)   5079 × 7093

9)   9999 × 808

10) 5898 × 7809

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ABACUS - DIVISION

I. GENERAL RULES :

a) Three terms - divisor, dividend and the quotient are used in a division problem. The number that does the dividing is called the DIVISOR. The number into which it is divided is called the DIVIDEND. The answer obtained by DIVIDEND / DIVISOR is known as the QUOTIENT. For example, take the problem 63÷7. Here 7 is the divisor, 63 is the dividend and the value 63÷7 = 9 is the quotient.

b) Take the example 72÷7. Here 7 is the divisor, 72 is the dividend. 72 = (7 × 10) + 2. Here, the quotient of 72÷7 is 10 and the remaining value '2' is known as the REMAINDER.

c) In division problems, the divisor of the problem should be set at the extreme left of the abacus and the dividend at the extreme right of the abacus. Please note that no column is added for division problems. The digits of the dividend should be placed in the respective units, tens, and hundreds columns and so on in the right side of the abacus.

d) While dividing, you need a place for setting the quotient. If the divisor is equal to or smaller than the first digit of the dividend, skip one column to the left of the dividend and set the quotient. If the quotient is a multiple digit number, you need additional columns for setting the other digits of the quotient. In such cases, you will be setting the digit of the highest column first and the remaining digits should be placed to the consecutive right of the previous digits.

e) If the divisor is greater than the first digit (highest column) of the dividend, there is no need for skipping one column to the left of the dividend. You can place the quotient to the immediate left of the dividend.

f)   When the setting of the dividend and the quotient are correct, the remainder of the division problem, if any, will appear in the appropriate units, tens, hundreds columns and so on in the abacus.

II.   SHORT DIVISION :

A.   *Example : 912 ÷ 4*

a)   Set the divisor 4 in the extreme left of the abacus.

b)   Set the dividend 912 in the extreme right of the abacus. The number 9 should be placed in the hundreds column, 1 in the tens column and 2 in the units column.

c)   Here the divisor 4 is smaller than the first digit of the dividend 9. Hence skip one column to the immediate left of the dividend and place your quotient.

d)   Place your left hand on the divisor 4. Keep your right hand on the 9 of the dividend 912. Ask : how many 4's are in 9? By saying 2, skip one column to the immediate left of the number 9 and set the quotient 2 using your left hand. Now by saying 2 × 4 = 8, subtract 8 from the number 9. Now you have 1 in the hundreds column, 1 in the tens column and 2 in the units column.

e)   Now take 1 of the 112. Note that the divisor 4 is greater that the first digit of the dividend 112. Hence, you need not skip a column to the left for setting your quotient at this stage of division. The quotient at this stage can be placed to the immediate left of the dividend, which is, the immediate right of the first quotient 2 and immediate left of 1 of 112.

f)   Now ask : how many 4's in 11? There are 2. Set the quotient to the immediate right of the other number 2 which has already been set as the first digit of the quotient. By saying 2 × 4 = 8, subtract it from 11 of the dividend. Place the number 2 to the immediate left of the dividend which is the immediate right of the number 2 of the quotient already set. At this stage, we have 32 left in the dividend. Now ask : how many 4's are in 32? By saying 8, set that in the

immediate right of the already set quotient 23. Now subtract 32 from the dividend. There is no remainder left.

g) Now you are left with the number 228 which is the required answer. That is, 912 ÷ 4 = 228, the quotient.

B.   Example : 384 ÷ 6

a) Set the divisor 6 in the extreme left of the abacus.

b) Set the dividend 384, the number 3 must be placed in the hundreds column, 8 in the tens column and 4 in the units column.

c) Here, the first digit of the dividend is 3 which is less the than the divisor 6. Hence, the quotient should be placed to the immediate left of the dividend.

d) Place your left hand on the divisor 6. Place your right hand on the number 3 of the divisor. Since 3 is smaller than 6, move your right hand and check the number. It is 8. Now consider 38. Now ask : how many 6's in 38? By saying 6, place the 6 in the immediate left of the number 3 using your left hand. Now move your hands to the right by saying 6 $\times$ 6 = 36, that is to be subtracted from 38. Clear 3 of the 3 in the hundreds column, 6 of the 8 in the tens column. Now you have 2 in the tens column and 4 in the units column. Ask : how many 6's in 24? By saying 4, set the number 4 in the immediate right of the higher digit of the quotient 6 which has already been set. By saying 4 $\times$ 6 = 24, subtract 2 from the 2 and 4 from the 4. Now you have nothing left as remainder. You will find the quotient as 64. That is 384 ÷ 6 = 64.

e) For checking your answer, multiply 6 with 64. If you get 384, your answer is correct. Otherwise do it once again.

EXERCISE

1)	756	÷	4
2)	575	÷	5
3)	625	÷	6
4)	838	÷	2
5)	9098	÷	3
6)	674	÷	4
7)	238	÷	3
8)	839	÷	7
9)	1329	÷	9
10)	3612	÷	4

ABACUS - LONG DIVISION

Example : 3588 ÷ 46

a)   Set the divisor 46 in the extreme left of the abacus. Note that the divisor has two digits.

b)   Set the dividend in the extreme right of the abacus. That is, 3 in the thousands column, 5 in the hundreds column, 8 in the tens column and 8 in the units column.

c)   Here, the first digit of the divisor 4 is greater than the first digit of the dividend, that is 3 of the 3588. Hence, the quotient should be set to the immediate left of the dividend.

d)   When the divisor is a multiple digit number, we have to take an ASSUMED QUOTIENT. See that it is assumed. If your judgement is correct you will get assumed quotient as the actual quotient. If it is not correct, your calculation itself will show you that alteration of the assumed quotient would be necessary. Let us illustrate this idea by solving this problem.

e)   In selecting the assumed quotient, you can follow a simple technique which holds good most of the time. See the first digit of the dividend. If it is less than the first digit of the divisor, consider the first two digits of the dividend. In this problem, it is 35. See the first digit of the divisor. It is 4. *Always add 1 mentally to the first digit of the divisor.* Here the total is 5. Now ask : how many 5's in 35? You get 7. Seven can be treated as the assumed quotient. Most of the time, the assumed quotient selected in this manner becomes the actual quotient.

f) Now multiply 7 with 4 of the divisor 46. $7 \times 4 = 28$. By saying this, subtract 28 from the 35 of the 358. Now you are left with 78 in 358. Say $7 \times 6 = 42$ and subtract 42 from the 78. You are now left with 36 which is smaller than the divisor 46. Hence, your assumed quotient is the actual quotient. In the abacus, you will now find 3 in the hundreds column, 6 in the tens column and 8 in the units column.

g) The divisor is 46 and you have 368 in the dividend. Select the assumed quotient as 8. Multiply 8 by the 4 of the dividend. By saying $8 \times 4 = 32$, clear 3 from the 3 of the hundreds column and 2 from 6 of the tens column. You are left with 4 in the tens column and 8 in the units column. Now multiply 8 with the 6 of the divisor. By saying $8 \times 6 = 48$, clear the 48, the number left in the abacus.

h) The quotient is 78.

   Note : In selecting the assumed quotient :

   i) In the previous problem, if you had selected 6 as the assumed quotient, what would have happened? Don't worry. Carry on with the problem. Consider 358 of the dividend. As in the case of multiplication, multiply the divisor 46 and the assumed quotient 6. You get 276. When 276 is subtracted from 358, you get 82 as the remainder which is greater than the quotient. DON'T RESET THE PROBLEM, Ask : How many 46's in 82. By saying 1, you add the 1 with the already set assumed quotient 6, and thus ALTERING the quotient as 7. Now multiply 1 by the divisor 46. By saying 46, subtract 46 from 82. The remaining number is 36. Now consider 368 and proceed as in the previous case. Here also the assumed quotient is selected as mentioned earlier.

   ii) When you select 8 as the assumed quotient, what will happen? Your calculation will point out your mistake! You are considering 358 of the dividend for calculation at the first stage. Now multiply 46×8. That is, multiply $4 \times 8 = 32$ and subtract that value 32 from 35. You are left with 38 in the abacus. Multiply 6 with 8. The value 48 cannot be subtracted from 38. Hence the assumed quotient is wrong. Reduce 1 (number) from the assumed quotient. RESET the dividend and proceed. When you are correct, the abacus will show the correct result.

EXERCISE

1)	650	÷	25
2)	9250	÷	50
3)	6868	÷	68
4)	1748	÷	38
5)	8832	÷	92
6)	9016	÷	98
7)	2756	÷	53
8)	6006	÷	77
9)	8649	÷	93
10)	4692	÷	68

ABACUS DIVISION - QUOTIENT AND REMAINDER

Example : 4988 ÷ 28

a)   Set the divisor 28 at the extreme left of the abacus.

b)   Set the dividend at the extreme right of the abacus.

c)   Add 1 with the first digit of the divisor (mentally). You get 3. Now ask how many 3's in 4? It is 1. 1 may be your assumed quotient. Set the assumed quotient to the left of the first digit of the dividend by skipping the immediate left column. This is done because 2, the first digit of the divisor is less than 4, the first digit of the dividend. Now you can start multiplying. By saying 1 × 2 = 2, subtract 2 from 4. By saying 1 × 8 = 8, subtract the 8 from 9. You are left with 21.

d)   Next, consider 218. This is a very typical case. If you look at the divisor and the dividend, the first digits are exactly the same i.e., 2. Thus, you are tempted to set the quotient by skipping one column to the left. If you skip one column to the left, you have number 1 in the quotient which has already been set. Are you going to add your next digit of the quotient with that 1? ABSOLUTELY NOT. In such cases, do we follow any other procedure? YES.

e)   For setting the first digit of the quotient, you considered the first two digits of the dividend into which the divisor is divided. In that case, you considered only the first digit of the divisor. Now you have to consider three digits of the divisor into which the divisor should be divided. When you consider three digits of the divisor, see

whether or not the first two digits are smaller than the first two digits of the dividend. If the divisor is bigger, do not leave a column. Otherwise leave a column. Most of the time, you need not skip the columns after the setting of the first digit of the quotient.

f) In this problem, consider the divisor 28 and 21 of the dividend 218. Since 28 is greater than 21, do not leave a column. The quotient should be set to the immediate right of the already set quotient 1.

g) Now consider 2 of the divisor. Add 1 mentally. Ask how many 3's in 21? By saying, 7, set the 7 to the immediate right of the quotient 1. Now multiply 7 × 28 and subtract that value from 218. You will be left with 22.

h) Now consider 228. By the above mentioned method, set 7 to the immediate right of 17, the quotient already set in the abacus. Now multiply 7 × 28 and subtract the value 196 from 228. You get the remainder as 32 which is greater than the divisor.

i) Don't worry. Ask : how many 28's in 32. By saying 1, add (no new setting) that with the 7 which is set just now, that is the third digit of the quotient. Now it becomes 8. This means your last digit of the quotient is altered as 8 instead of 7. Now subtract 28 from 32. You get 178 as the quotient and 4 as REMAINDER.

j) Note that the numbers of the remainder appear in the appropriate columns.

EXERCISE

1)	4552	÷	48
2)	5436	÷	54
3)	6844	÷	37
4)	7453	÷	38
5)	8436	÷	43
6)	9834	÷	24
7)	2832	÷	23
8)	5428	÷	24
9)	9009	÷	34
10)	8819	÷	26

ABACUS - DECIMAL ADDITION

a) The procedures for addition of decimals are same as those in the case of addition of whole numbers. The only new aspect here is the placement of the decimal point in the abacus.

b) The dots of the separation bar of the abacus can be used for separating the whole number and the decimal part of the given number.

c) For example, consider 32.67. Here, the decimal part of this number has two digits. You need only two places for setting this. Hence select the first dot of separation bar. The 2 of the whole number can be set in the thousands column and the 3 in the ten thousands column. The columns for whole numbers must be set to the left of the decimal point.

d) The setting of the decimal part should start from the decimal point in the right side. THE WHOLE NUMBER SHOULD BE SET IN THE LEFT SIDE OF THE DECIMAL POINT AND THE DECIMAL PART IN THE RIGHT SIDE OF THE DECIMAL POINT.

e) Suppose you want to set the number 45.8972. Here, the decimal part has four digits. Hence, select the second dot of the separation bar as the decimal point. Here, the 8 of the decimal part should be set in the 6th column from the right extreme of the abacus, 9 in the fifth, 7 in the fourth and 2 in the third columns. The 5 of the whole number is set in the immediate left to the decimal point, and the 4 in the eighth column.

f) **Example : 32.79 + 425.692**

First set the number 32.76 in the abacus. Here you have only 3 digits after the decimal point. Hence, fix the decimal point after the first dot in the right side of the abacus.

g) Add the number 425.692 as you do in the normal addition. In the first number, you do not have hundreds column. Set the number 4 (hundreds column) of the second number which should be set in the abacus. **Always remember that addition should be made only with the corresponding digits. This is true for decimals also. For convenience, zero can be added before or after the number to level decimal points in both the numbers. For example, in the above sum, 32.76 can be treated as 032.760.**

In problems dealing with decimal points, the digits will not appear in the respective units, tens, and hundreds columns of the abacus since the dots of the separation bar are used for setting the whole number and decimal portions of the numbers. You should remember that the positions of the numbers should be treated only with respect to the dots used for setting the whole number and the decimal portion. This is the major difference between normal addition and decimal addition.

EXERCISE

1)	43.56	+	56.987
2)	567.001	+	34.78
3)	651.8	+	23.8
4)	43.00001	+	32.65432
5)	49126.543	+	23.4567
6)	23.4951	+	41.90
7)	43.8905	+	45.76
8)	32.76	+	457.32
9)	23.87	+	41.76
10)	41.888	+	2.42

ABACUS - SUBTRACTION OF DECIMALS

Abacus - Subtraction of decimals :

The procedures followed in setting the decimal number for subtraction are same as of those in addition. After setting the number, usual subtraction procedures as in the case of subtraction of whole numbers are to be followed:

EXERCISE

1)	42.789	–	34.99
2)	567.001	–	34.78
3)	651.8	–	23.8
4)	43.00001	–	32.65432
5)	49126.543	–	23.4567
6)	56.75	–	45.76666
7)	45.8905	–	45.76
8)	457.32	–	32.76
9)	41.76	–	23.87
10)	41.888	–	2.42

ABACUS - MULTIPLICATION OF DECIMALS

Before multiplication, the multiplier and the multiplicand should be converted into whole numbers. For example, 32.76 × 234.5. Here, the multiplier has two digits in the decimal part and the multiplicand has only one digit in the decimal part. For converting the multiplier into a whole number (note that the multiplier has the maximum digits in the decimal part), multiply the number 32.76 × 100 which is 3276. Now you have to convert the multiplicand into a whole number. Multiply it by 10. **In such cases, both the multiplier and the multiplicand need not be multiplied by the same number in decimal multiplication. Hence, multiply the multiplicand 234.5 by 10 and make it 2345.** Now treat the problem as 3276 × 2345. Apply the usual procedures for multiplication. You will get the value as 7682220.

Now look at the original decimal values. Your multiplier and the multiplicand together have 3 places in the decimal part. Leave three numbers of the right and insert the decimal point to get the required value. The three numbers in the right form the decimal part. In this problem your value will be 7682.220.

EXERCISE

1)	3.24	×	45.7	6)	32.87 × 21.111
2)	3.54	×	87.9	7)	543.8 × 213.8
3)	76.101	×	32.98	8)	65.1 × 32.99
4)	65.87	×	34.67	9)	76.01 × 2.453
5)	43.8	×	32.909	10)	3.393 × 1.213

ABACUS – DIVISION OF DECIMALS

Convert the dividend and the divisor into whole numbers. While converting the divisor and the dividend, both the numbers should be multiplied by the same number (10 or 100 or 1000). The decimal multiplication procedure will not apply here.

Consider this example : $\dfrac{62.5}{1.25}$

Here the divisor has two decimal digits. Hence multiply both the divisor and the dividend by 100 to convert them as whole numbers.

You will get 6250 and 125 respectively because you multiplied both the dividend and the divisor by 100 (since you have a maximum of two decimal digits in the divisor). Follow the normal procedures for getting the quotient. After dividing you get 50 as the answer.

EXERCISE

1)	7.25	÷	1.2	6)	422.5 ÷	0.65
2)	15.6	÷	1.3	7)	196 ÷	0.16
3)	22.5	÷	1.5	8)	888.1 ÷	0.09
4)	10.5	÷	1.5	9)	99.9 ÷	0.9
5)	12.1	÷	1.1	10)	1000 ÷	12.5

ABACUS – FRACTION ADDITION

I. GENERAL RULES :

a) Numerals such as $\frac{3}{5}, \frac{7}{10}$ etc., are known as fractions. In these fractions, the numbers 3 and 7 are known as numerators and numbers 5 and 10 are denominators. Numerals like $4\frac{6}{7}$, $5\frac{3}{4}$ etc., are known as complex fractions.

b) *Least Common Multiple :* Consider two fractions, $\frac{4}{5}$ and $\frac{7}{9}$. Here 5 and 9 are the denominators of the respective fractions. What is the common number into which both the numbers 5 and 9 could be divided without leaving a remainder? 45, 90, 135 etc., are some common denominators. In addition and subtraction of fractions, it is always suggested to use the smallest common denominator which is technically known as the Least Common Multiple.

c) In addition and subtraction of fractions, the abacus is divided into three sections, one for setting the whole number, the second for setting the numerator and the third for setting the denominator.

d) In fraction, the common denominator is set in the extreme right. Three columns - the units, tens and hundreds columns are allotted for setting the denominator. The fourth, fifth and sixth columns from the right side of the abacus are allotted for setting the numerator. The seventh, eighth and ninth columns from the right extreme of the abacus are allotted for setting the whole number. That is, the numerator is set between the whole number and the denominator.

e) Please note that only three columns are allotted for the denominator. The same procedure applies in the case of numerator and the whole number too. If your

denominator exceeds three digits, move the placement of the respective divisions according to your interest. You must note that the calculations do not run into one another.

II. ADDITION OF FRACTIONS :

Example : $5\dfrac{6}{7}+7\dfrac{4}{5}$

a) Calculate the common denominator for the fractions. The denominator of $\dfrac{6}{7}$ is 7 and $\dfrac{4}{5}$ is 5. For 7 and 5, 35 is the least common denominator.

b) Set the denominator (common) in the extreme right of the abacus. The unit digit of the number should always be set in the extreme right of the particular section. That is, place the 3 of the common denominator in the tens column and the 5 in the units column.

c) What is the whole number of the first fraction $5\dfrac{6}{7}$? It is 5.   Set five in the appropriate column. That is, set the number in the section meant for whole number, i.e, in the 7th column from the extreme right side of the abacus. With this add the whole number of the second fraction, that is, you have to add 7 with 5, making the whole number into 12.

d) Now take the fraction part of the first number. It is $\dfrac{6}{7}$. Set the numerator 6 in the extreme left of the abacus. Now skip one column to the right of it and set the denominator 7. There is no need to set the number in the abacus if you are able to remember the same. However, it is usually preferred to set the number for the purpose of reference.

e) By keeping your left hand on the 7 and right hand on the common denominator 35, ask : how many 7's in 35? By saying 5, you clear the 7 and set 5. With this 5, multiply the numerator 6. By saying $6\times5 = 30$, you have to set the 30 in the section meant for the numerator. That is, the 0 of the 30 should be set in the fourth column and 3 in the fifth column from the extreme right of the abacus.

f) Now clear the numbers set in the extreme left.

g) After clearing, set the numerator of the second fraction in the extreme left of the abacus. That is, your numerator of the second fraction 4 should appear in the first column from the extreme left and the denominator 5 in the third column from the extreme left.

h) By placing your left hand on the 5 and right hand on the denominator 35, ask : how many 5's in 35? By saying 7, clear the 5 and set 7 in the same column. Now multiply 7 and 4. By saying $7 \times 4 = 28$, add the number 28 with the value 30 in the numerator section. Add 2 with 3 and 8 with 0. You get the value 58.

i) At this stage, check the numerator and the denominator sections. If the value of the numerator section is less than the value of the denominator section, leave it. If the value of the numerator section is greater than the value of the denominator section, you have to do some additional calculations.

j) In the result, you should get a fraction in such a way that the numerator is always less than the denominator. In this problem, your numerator value is 58 and the denominator value is 35. Now ask : how many 35's in 58? By saying 1, add the number 1 (which is nothing but $\frac{35}{35}$) with the whole number set already in the whole number section, thus making the whole number as 13. At the same time, the due 35 must be subtracted from the numerator. Now you have 23 in the numerator and 35 in the denominator which is in the right form.

k) At this stage, check all the three sections. You have 13 as whole number, 23 as numerator and 35 as denominator. The answer is $13\frac{23}{35}$.

EXERCISE

1) $\dfrac{4}{5}$ $+$ $\dfrac{7}{8}$

2) $\dfrac{9}{10}$ $+$ $\dfrac{12}{13}$

3) $\dfrac{14}{15}$ $+$ $\dfrac{4}{5}$

4) $2\dfrac{5}{8}$ $+$ $3\dfrac{4}{7}$

5) $12\dfrac{7}{9}$ $+$ $14\dfrac{2}{7}$

6) $24\dfrac{13}{16}$ $+$ $27\dfrac{1}{4}$

7) $235\dfrac{4}{9}$ $+$ $98\dfrac{6}{7}$

8) $135\dfrac{3}{4}$ $+$ $98\dfrac{6}{7}$

9) $44\dfrac{21}{25}$ $+$ $45\dfrac{19}{20}$

10) $59\dfrac{15}{16}$ $+$ $65\dfrac{14}{15}$

11) $76\dfrac{27}{50}$ $+$ $80\dfrac{2}{5}$

12) $34\dfrac{11}{14}$ $+$ $23\dfrac{11}{28}$

13) $76\dfrac{7}{8}$ $+$ $156\dfrac{11}{12}$

14) $45\dfrac{9}{12}$ $+$ $51\dfrac{7}{9}$

15) $1190\dfrac{1}{9}$ $+$ $124\dfrac{7}{11}$

ABACUS – FRACTION SUBTRACTION

a) The procedures followed in fraction addition are to be followed in fraction subtraction also. Here, the numerator and the whole number of the second fraction should be subtracted from the corresponding numerator and the whole number of the first fraction.

b) Consider the example : $9\frac{5}{8} - 6\frac{11}{12}$.

c) Set the whole number, numerator and the denominator of the first fraction in the respective columns.

d) See the denominators of the two fractions. They are 8 and 12. What is the lowest common denominator of these two? It is 24. Now set the common denominator in the right extreme of the abacus.

e) Subtract the whole number of the second fraction from the whole number of the first fraction. That is 9–6=3. You are left with 3 in the whole number column.

f) Now take the first fraction. It is $\frac{5}{8}$. Set the numerator 5 in the extreme left of the abacus; skip one column and set the denominator 8. By keeping your left hand on the 8 and right hand on the common denominator 24, ask : how many 8's in 24? By saying 3, clear the eight and set 3. Now multiply 3 and the numerator 5. By saying 15, set the 15 in the numerator column. That is, set the 1 in the fifth column and 5 in the fourth column respectively. Clear the first fraction which has been set in the extreme left of the abacus.

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g) Now take the second fraction. Set the numerator 11 of the fraction in the extreme left, leave one column and set the denominator 12. By keeping your left hand on the 12 and right hand on the common denominator 24, ask : how many 12's in 24? By saying 2, clear the 12 and set the 2. Now multiply 2 and 11. This value $2 \times 11 = 22$ should be subtracted from the numerator already set in the abacus.

h) Check the value in the numerator. It is 15. You have to subtract 22 from this according to your problem. This is not possible. In this case (i.e., when the numerator to be subtracted is greater than the available numerator in the abacus), look at the common denominator. That value should be added to the numerator before the subtraction. In this case, you have to add 24 with 15, thus making it 39. How did you get this? This is $\frac{24}{24} + \frac{15}{24}$. Please note that there is no change in the denominator.

i) You have added $\frac{24}{24} = 1$. This number 1 should be subtracted from the whole number column. Now you have 2 in the whole number column.

j) After subtracting this number 1, you have 39 (24 + 15) in the numerator. Now the value 22 of the second fraction can be subtracted from the numerator thus leaving 17 in the numerator.

k) Check the numbers now. You have 2 in the whole number column, 17 in the numerator and 24 in denominator.

l) Your answer is $2\frac{17}{24}$. Here the numerator 17 and the denominator 24 do not have a common divisor. Hence, the fraction part cannot be simplified further.

> **Note :**
> In fraction addition, when the numerator value is greater than the denominator value, subtract the denominator value from the numerator value. To compensate this add 1 in the whole number. In fraction subtraction, when the numerator value of the first fraction is less than the numerator value of the second fraction, add the value of the denominator with the numerator value of the first fraction and subtract 1 (one) from the whole number.

EXERCISE

1. $5\frac{6}{7}$ – $3\frac{2}{5}$

2. $7\frac{4}{7}$ – $5\frac{5}{7}$

3. $10\frac{2}{3}$ – $7\frac{4}{5}$

4. $13\frac{11}{15}$ – $8\frac{4}{5}$

5. $18\frac{12}{15}$ – $12\frac{3}{8}$

6. $22\frac{7}{8}$ – $12\frac{8}{9}$

7. $34\frac{21}{35}$ – $21\frac{4}{5}$

8. $35\frac{8}{9}$ – $32\frac{7}{12}$

9. $342\frac{45}{46}$ – $43\frac{20}{23}$

10. $325\frac{4}{9}$ – $9\frac{11}{18}$

11. $54\frac{7}{9}$ – $45\frac{99}{100}$

12. $4333\frac{23}{30}$ – $101\frac{39}{45}$

13. $455\frac{2}{5}$ – $43\frac{8}{15}$

14. $23\frac{7}{12}$ – $11\frac{8}{9}$

ABACUS – FRACTION MULTIPLICATION

I.   GENERAL RULES :

a)   In fraction multiplication, the number should be converted into improper fraction before multiplication. For example, the improper fraction of $5\frac{3}{4}$ is $\frac{23}{5}$. How did you get this? First, the whole number and the denominator of the fraction are multiplied. The numerator is then added with this value. This total value as the numerator and the original denominator value are presented in the form of a fraction. This fraction is known as improper fraction. In improper fraction, the numerator is always greater than the denominator.

b)   Two numbers are taken at a time for multiplication. After converting the given fractions into improper fractions the numerator of the first improper fraction is multiplied by the numerator of the second improper fraction. Similarly, the denominator of the first improper fraction is multiplied by the denominator of the second improper fraction.

c)   After the multiplication is through with this process, the value should be converted into a proper fraction by dividing the numerator by the denominator. In doing so you can calculate the whole number, numerator and the denominator.

d)   For multiplication of small fractions involving single digit numerator and denominator, one abacus will be sufficient. But multiplication process becomes complicated with one abacus when fractions having multiple digit numerators and denominators are multiplied. In this case, the placement of multiplied value in the abacus will be difficult.

e) Two alternatives are possible for solving this problem.

i) Two abacuses may be used by the child, one for calculations and the other for recording the important steps of the calculations and the result.

ii) One abacus can be used for calculations and the child can record his answers in Braille using a Braille sheet and the Braille slate. Here the child should record all the different steps of the calculation.

f) The above alternatives are equally good. But the second one is time consuming whereas the first method is faster.

g) Once the fraction is converted into an improper fraction, the multiplication of the numerators is just like the normal multiplication. Similar procedures are to be followed in the case of denominators.

h) Memory of multiplication tables is very essential for converting the proper fraction into an improper fraction.

II. MULTIPLICATION WITH TWO ABACUSES :

Example : $5\dfrac{2}{12} \times 12\dfrac{11}{13}$

a) With the first abacus, change the first fraction into improper fraction. For this,

i) *Multiply 12 and 5 using regular multiplication procedures.*

ii) *With the multiplied value 60, add 2 of the numerator.*

b) Now set the numerator of the improper fraction $\dfrac{62}{12}$ in the second abacus. In doing so, the numerator should be set in the left and the denominator in the right side of the abacus. Set the 62, 6 in the extreme left and 2 in the immediate right of the number 6 in the left side of the abacus. The denominator 12 should be set in the extreme right as you set the number in addition. That is your 2 should be set in the units column and the 1 in the tens column.

c) Now clear the number set in the first abacus. You are ready for your second fraction. Convert $12\frac{11}{13}$ into improper fraction according to the procedures mentioned above. You will get the improper fraction as $\frac{167}{13}$.

d) Now you have to set this value in the second abacus. The numerator and the denominator of the first fraction are already set in that abacus. Where do you set the values of the second fraction? The numerator of the second fraction should be placed to the right side of the numerator of the first fraction by leaving two columns in between. In this case, you have to set your 167 from the fifth column in the left side of the abacus. The denominator of the first fraction should be set in the sixth and fifth columns from right leaving two columns in between the denominators.

e) Now clear the numbers of the first abacus. In the second abacus you have 62 and 167 in the numerator side and 12 and 13 in the denominator side.

f) Now a new idea "CANCELLATION" is introduced. The numerators and denominators can be made smaller by a common divisor, if possible so that the calculations would be easier. For clarity, the calculations can be done in the first abacus which is free now. Consider, number 62 – the numerator and 12 – the denominator. 2 is a common divisor. Both 62 and 12 can be divided by 2 thus making them 31 and 6 respectively. Now you have 31 and 167 in the numerator side and 13 and 6 in the denominator side.

g) CANCELLATION SHOULD BE DONE ONLY BETWEEN THE NUMERATOR AND DENOMINATOR. ON ANY ACCOUNT, CANCELLATION SHOULD NOT BE MADE BETWEEN THE NUMERATOR AND NUMERATOR OR DENOMINATOR AND DENOMINATOR.

h) Now, take 31 and 167. Treat this as 31×167. Now, this is in the form of your usual multiplication. Using your first abacus, solve the problem. The answer is 5177.

i) Clear the numerator side of the second abacus. Set the number 5177 in the extreme left.

j)   Clear the first abacus. Set the denominator values as 6×13. The answer is 78.

k)   Clear the denominators of the second abacus and set 78 in the denominator side. Now you are left with 5177 in the numerator side and 78 in the denominator side.

l)   Clear the first abacus. Set 5177 as dividend and 78 as the divisor. Divide the number. Normal division procedures have to be followed. You will get 66 as the quotient and 29 as the remainder.

III.   MULTIPLICATION USING ABACUS AND SLATE AND STYLUS :

Consider the previous example itself:

1.   Make the fractions into improper fractions using the abacus.

2.   Write the values in Braille sheet. This is for your reference (child's reference).

3.   Cancellation can be made, if any.   While using Braille you may not set the number in the abacus for making the cancellations. This can be done in memory also.

4.   Now read the values of the numerators and denominators which are written.

5.   Multiply the numerators using the abacus.

6.   Write your answer ; clear the abacus now.

7.   Multiply the denominators using the abacus.

8.   Write your answer ; clear the abacus now.

9.   Set the value of numerator as dividend and the value of denominator as divisor; divide now.

10.   Write the answer – the quotient and the remainder.

EXERCISE

1. $\dfrac{4}{5} \times \dfrac{7}{8}$

2. $\dfrac{9}{10} \times \dfrac{12}{13}$

3. $\dfrac{14}{15} \times \dfrac{8}{45}$

4. $2\dfrac{6}{8} \times 3\dfrac{4}{7}$

5. $12\dfrac{7}{9} \times 14\dfrac{2}{7}$

6. $24\dfrac{13}{16} \times 27\dfrac{1}{4}$

7. $235\dfrac{4}{9} \times 98\dfrac{6}{7}$

8. $135\dfrac{3}{4} \times 98\dfrac{6}{7}$

9. $44\dfrac{21}{25} \times 45\dfrac{19}{20}$

10. $59\dfrac{15}{16} \times 65\dfrac{14}{15}$

11. $78\dfrac{27}{50} \times 80\dfrac{2}{5}$

12. $34\dfrac{11}{14} \times 23\dfrac{11}{28}$

13. $76\dfrac{7}{8} \times 156\dfrac{11}{12}$

14. $45\dfrac{9}{13} \times 51\dfrac{8}{9}$

15. $190\dfrac{1}{9} \times 124\dfrac{7}{11}$

ABACUS – DIVISION OF FRACTIONS

a) If you are through with fraction – multiplication, fraction - division will not be difficult.

b) Consider this example : $10\frac{4}{5} - 6\frac{2}{3}$.

c) First, change them into improper fractions. You will get the improper fractions as $\frac{54}{5}$ and $\frac{20}{3}$ respectively.

d) In multiplication, you multiplied the 54 and 20, the numerators; 5 and 3, the denominators and after dividing the numerator value by the denominator, you obtained the answer.

e) In this problem, you have to divide. Here improper fraction $\frac{54}{5}$ is the dividend and $\frac{20}{3}$ is the divisor.

f) Invert the divisor. Then $\frac{20}{3}$ will appear as $\frac{3}{20}$.

g) Now consider $\frac{54}{5}$ and $\frac{3}{20}$ as a new pair of fractions and apply the multiplication rules.

h) Multiply the numerators of the new pair of fractions. The value is 54 X 3 = 162. Multiply the denominators of the new pair of fractions. The value is 5 X 20 = 100.

i) Now divide the dividend 162 by the divisor 100. You will get 1 as the quotient and 62 as the remainder, i.e., $1\frac{62}{100}$. The fraction part $\frac{62}{100}$ can be simplified and you get $\frac{31}{50}$. The answer is $1\frac{31}{50}$.

j) You need not invert the answer again. To make the process of division simpler, you had inverted the divisor in the beginning itself.

EXERCISE

1. $6\frac{5}{7}$ ÷ $3\frac{2}{5}$

2. $7\frac{4}{7}$ ÷ $5\frac{5}{7}$

3. $10\frac{2}{3}$ ÷ $7\frac{4}{5}$

4. $13\frac{11}{15}$ ÷ $8\frac{4}{5}$

5. $18\frac{12}{15}$ ÷ $12\frac{3}{8}$

6. $22\frac{7}{8}$ ÷ $12\frac{8}{9}$

7. $34\frac{21}{35}$ ÷ $21\frac{4}{5}$

8. $56\frac{7}{11}$ ÷ $32\frac{8}{12}$

9. $35\frac{8}{9}$ ÷ $32\frac{7}{12}$

10. $342\frac{45}{46}$ ÷ $43\frac{20}{23}$

11. $325\frac{4}{9}$ ÷ $9\frac{11}{18}$

12. $54\frac{7}{9}$ ÷ $45\frac{99}{100}$

13. $433\frac{23}{30}$ ÷ $101\frac{39}{45}$

14. $455\frac{2}{5}$ ÷ $43\frac{8}{15}$

15. 23 7/12 ÷ 11 8/9

ABACUS – SQUARE ROOT

I.   GENERAL RULES :

a)   Consider the number 25. 25 can be written as 5x5. Here 5 and 5 are known as factors. Similarly 2, 12 are factors of 24 as 24=2×12. 3 and 8 are also factors of 24 as 3×8=24. What is the speciality of the first set? THE FACTORS ARE EQUAL. ONE OF THE EQUAL FACTORS OF THE NUMBER IS KNOWN AS THE SQUARE ROOT OF THAT NUMBER. Since the number 25 has two equal factors 5 and 5, the number 5 is called the square root of the number 25. 25 is known as the SQUARE of the number 5.

b)   Square roots for numbers can be found with the help of abacus. Suppose you are expected to find the square root of the number 2456. Leave the first three columns in the right extreme of the abacus and set the number from the fourth column. That is, you have to set the number 6 in the fourth column, 5 in the fifth column, 4 in the sixth column and 2 in the seventh column from right.

c)   If the square root does not have decimal values, the square of that number (square root) is termed as PERFECT SQUARE. 4,9,16,25,36,49,64 etc., are some of the perfect squares.

d)   For setting the square root of the number in the abacus, the rules followed in setting the quotient in a division problem are applicable. The square root value should be set in the left side of the given number.

e)   For finding the square root, the digits of the given number should be considered in groups. For example, in the problem 3465, the groups must be 34 and 65.

In the number 16783, the groups will be 1,67 and 83. In grouping, start from the right side of the number. If you are left with a single digit in the left, treat that as a group. These are clearly illustrated in the model exercises.

II. FINDING THE SQUARE ROOT OF A PERFECT SQUARE :

A. Example : 324

a) Set the number 324 in the abacus. That is, 3 must be set in the sixth column from the right, 2 in the fifth and 4 in the fourth column.

b) Now ask. What is the highest perfect square in 3. (You are grouping this number as 3 and 24). It is 1. Set the 1 by skipping one column to the left of 3. By saying 1×1, clear 1 from the 3. Now you are left with 224.

c) Double the first digit of the square root and place it in the left extreme of the abacus. That means, you have to set 2 in the extreme left of the abacus.

d) Now consider the 22 of the remainder. For setting the second digit of the square root, many ways are available.

- You ask : how many 2's in 22? It is 11. But you should not set two digits in the quotient (here square root) at a time. Hence consider the highest single digit number. It is 9. Set the 9 to the right of the first digit of the square root 1. You will get 19.

- Set the 9 also in the right side of the number 2 which is already set in the left extreme of the abacus.

- Now multiply the 9 of the 19 with the 29 and subtract the value from 224. Multiply 9×2 and subtract the value 18 from 22. You will get 4 as remainder and altogether, you are left with 44 in the abacus. Multiply $9 \times 9 = 81$. 81 cannot be subtracted from 44. That means, your assumed quotient 9 is too big.

- Reduce 1 and set the value as 8. Now reset the dividend part as 224. Multiply 8×28 and subtract from the 224. You will be left with no

remainder. That means, square root of 324 is 18 and 324 is a perfect square.

You will really feel that this is a long process. But this can be simplified.

IN THIS PROBLEM YOU ASSUMED A MAXIMUM OF 9 AS THE QUOTIENT. ALWAYS REDUCE 1 from this and set it as the quotient. Most of the time, this value will be true. In case you get more remainder, add 1 to the quotient and subtract the number from the quotient as in the case of division.

You will find this easy when you solve many problems.

B.   Example : 1296

a)   Group this as 12 and 96

b)   Highest perfect square in 12 is 9. Thus set the square root of 9 which is 3 in the left side of the number 12.

c)   Subtract 9 from 12. The remaining value is 396.

d)   Double the 3 and set the 6 in the extreme left. Now ask : how many 6's in 39. You get a maximum of 6. You can select this as the second digit of the square root. To be on the safe side, set 5 to the right of 6 and make it 65. Set 5 to the right of 3 and make it 35.

e)   Multiply 5 with 65 and subtract from 396. That is subtract $5 \times 65 = 325$ from 396. You get remainder as 71 which is greater than 65. Subtract 65 from 71. You are left with 6.

f)   Please look at the last digit of the square root. It is 6. The remainder is also 6. This is nothing but the difference between the multiplication of 5 of the 35 with 6, i.e., $5 \times 6 = 30$ and the multiplication of the new additional number in the square root, i.e., $6 \times 6 = 36$. This happens only when your assumed quotient in the case of square root is less than that of the actual quotient. When the actual quotient is same as the assumed quotient, this will not happen.

g)   Hence it has been compensated. Thus, leave the remainder. You get 36 as the square root of the number 1296.

C.   Example : 4225

a)   Set the number 4225. Ask : What is the highest perfect square in 42. By saying 36, set the square root of 36 in the left of 4225.

b)   Subtract 36 from 42. Hence you will be left with 625. Now double the 6 and set in the extreme left.

c)   Now take 12 of the answer and 625 which is to be divided. Take the first digits. How many 1's in 6. Though it is 6, reduce 1, and therefore, it will be 5.

d)   Set the 5 to the right side of the first digit of the square root and make it 65. Set the 5 to the right and make it 125. Now multiply 5×125 and subtract it from 625. You will not get any remainder. Thus the assumed quotient 5 is the actual quotient.

e)   That is, the square root of 4225 is 65.

EXERCISE

1.	144	6.	225
2.	256	7.	625
3.	484	8.	729
4.	841	9.	961
5.	529	10.	121

ABACUS - SQUARE ROOT OF MULTIPLE DIGIT NUMBERS

A.   **Example : 103041**

a)   Group the digits as 10, 30, 41.

b)   9 is the highest perfect square in 10. Hence set the square root of 9, i.e., 3 and subtract 9 from 10. You are left with 13041.

c)   Double the number 3 and set that 6 in the extreme left of the abacus.

d)   Consider 13 of the 130, as 1 is smaller than 6. Ask : how many 6's in 13. You get your answer as 2. In case of small numbers you need not have an assumed quotient. Here you are clear that there are two 62's in 130. Thus straight away set the number 2 to the right of the first digit of the square root and to the right side of the number 6 which is set in the extreme left of the abacus. [In the case of big numbers like 9, 8 etc., you are expected to reduce 1 (number) always for calculations. In the case of small numbers, you can set the quotient directly].

e)   Multiply 2×62. Subtract the value 124 from 130. You are now left with 641 in the abacus. Look at the square root now.

f)   In this problem, you are going for a third digit in the square root. Now clear the 62 which is set in the left extreme.

g)   The first two digits of the quotient are 3 and 2. Treat this as 32. Double the number. Set the number 64 in the left extreme.

h)   Now consider 64 and 641. Look at the first digit of the dividend and the divisor. Here also you get small numbers. Ask : how many 6's in 6. By saying 1, set the 1 to the right of the square root and to the right of number 64. Multiply 1 with 641 and subtract that from the 61. No number is available in the abacus as remainder.

i)   You will get the answer as 321.

EXERCISE

1.	22415	6.	427716
2.	7885	7.	308025
3.	3025	8.	974169
4.	14641	9.	7921
5.	42025	10.	5929

ABACUS - SQUARE ROOT OF IMPERFECT SQUARE

SQUARE ROOT OF IMPERFECT SQUARE:

A. **Example : 69744**

a) Set the number in the abacus. In this number, the groups will be 6,97,44. In dealing with five digit number, it is suggested to take the first three numbers as a whole for finding the square root.

b) Take 697. Ask : what is the highest perfect square in 697. It is 676. Set the square root 26 to the left of this number and subtract 676 from 697. You will be left with 2144 in the abacus.

c) Double the square root of the first set 26 and set that in the left extreme of the abacus. You have to set 52 at the left extreme.

d) Now consider the divisor 52 and the dividend 2144. How many 5's are in 21? There are 4. Set this 4 to the right of the square root of the first two digits. You will get 524 at the left extreme. Now multiply 4×524 and subtract the value from 2144. You will get the remainder 48.

e) Now you have to go for your decimal part of the square root. Add one zero to the dividend 48 and make it 480.

f) For divisor, you can double the present square root 264. If you take 528 itself, the number will not affect the value because we are dealing only with the decimals. Since 480 is less than 528, your first decimal digit of the square root will be 0. Now add one more zero to the dividend. Now you get 4800.

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You ask : How many 500's in 4800. By saying 9, place the value to the right side of the present square root.

g) You will get the square root as 264.09.

B. Example : 75832

a) Set the number 75832 in the abacus.

b) Ask : What is the highest perfect square in 758? By saying 729, set the square root 27 to the left of the number and subtract 729 from 758. You are left with 2932 in the abacus.

c) Now double the square root and set it in the left extreme. The value is 54.

d) Now consider the divisor 54 and the dividend 2932. Now ask : how many 5's in 29. By saying 5, set the digit to the right side of the present square root. You will get the number as 275. Set this 5 to the right side of the 54. It will become 545. Multiply 5×545. The value is 2725. Subtract this from 2932. You will be left with 207.

e) Now you have to start with your decimal part of the square root. Add one zero to the remainder. It will become 2070. You ask : how many 550s (round figure) in 2050. Approximately 4. Set this 4 to the right side of the present square root.

f) The value is 275.4

EXERCISE

Find the square root of the following numbers :

1.	7884	6.	7885
2.	22415	7.	34342
3.	5638	8.	9876
4.	65423	9.	56431
5.	5454	10.	43088

ABACUS – PERCENTAGE

a) By percentage, we mean the hundredths. For example 56% means 56 out of 100. The notation "%" is used for percentage.

b) The percentage can be represented in forms of fractions and decimals.

Eg. : 25% can be denoted by the fraction $\frac{25}{100} = \frac{1}{4}$. It can also be given as $\frac{25}{100}$ = 0.25.

c) For finding the percentage value, you can use either the fraction procedures or the decimal procedures.

d) Finding the percentage value through decimal procedures :

Example : 35% of 567

- Convert the percentage into a decimal. The value will be 0.35.

- Now, set the 35 in the left extreme of the abacus. Now, the problem should be treated as 35×567.

- The normal procedures used for the multiplication of whole numbers should be followed at this stage.

e) Multiply the numbers and you will get the value as 19845.

- Since the number 35 has two decimal places, put the decimal point before the last two places at the right side of the number 19845. The answer is 198.45.

Example : 42.7% of 789.

i) Convert the percentage into a decimal. The value will be 0.427. Note that you have already one decimal in the given percentage value itself.

ii) Now treat the problem as 427×789. This is a normal multiplication problem. You will get the answer 336903. Since you have three digits in the decimal part, put the decimal point before the last three places at the right side of the multiplied value.

The answer is 336.903.

iii) Finding the percentage value through fraction procedures :

Example : 75% of 542.

- Convert 75% into a fraction. It is $\frac{75}{100} = \frac{3}{4}$. Here the numerator is 3 and the denominator 4.

- Set the values in the abacus as in the case of fraction multiplication. That is, set the value 3 in the left extreme and the value 4 in the right extreme of the abacus.

- Since this is a problem of fraction multiplication, the number 542 can also be treated as a fraction. You can treat this number as $\frac{542}{1}$, where 542 is the numerator and 1 is the denominator.

- Multiply 542 with 3 (both numerators) and multiply 4 with 1.

- The values will be 1626 and 4 respectively. Treat the number 1626 as dividend and 4 as the divisor. Divide now.

- The answer is 406.5. That is, 75% of 542 is 406.5.

EXERCISE

1. 5.2% of 176	6. 70% of 1234
2. 54.3% of 543	7. 88.9% of 329
3. 3.54% of 112	8. 10.1% of 311
4. 45% of 232	9. 48.2% of 1018
5. 25% of 5464	10. 32.32% of 245

Section 3

Use of Nemeth Mathematical Braille Codes
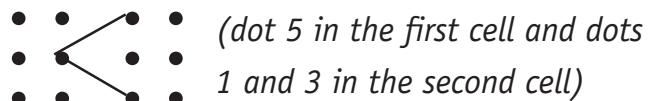
*Section on **"Braille Codes"** is treated as 25 modules and taught in **"25 hours"***

USE OF
MATHEMATICAL BRAILLE CODES
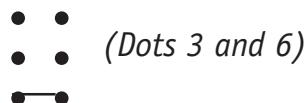
LEARNING NEMETH MATHEMATICAL BRAILLE CODE

Nemeth Mathematical braille codes are used in many parts of the world to teach mathematics to blind children.   The codes are close to visual configurations of particular symbols used with sighted children and therefore, applying a certain logic will help learn the codes without much difficulty.   For example, the symbol for less than (<) is written in braille using two cells resembling the visual configuration as

(dot 5 in the first cell and dots 1 and 3 in the second cell)

Though the six dots in a braille cell cannot present the visual configuration of every mathematical code, the sign, which is closest to the visual configuration is presented through the Nemeth code.   For example, the minus sign (–) is shown by a single cell configuration as

(Dots 3 and 6)

Therefore, one can look for visual configuration of the codes as clues to learn Nemeth Braille codes. The visually impaired person, who does not know the visual configuration of the signs may find it difficult to use this logic, but wherever possible, this logic may be used.

The purpose of using this logic is to avoid memorisation of the codes. In addition to the visual configuration, some structural configurations are also used in learning mathematical codes. For example, all shapes used in Geometry can be easily understood by the learner, whether visually impaired or sighted, by using a simple logic. For indicating shape, the code used is

⠪ (dots 1,2,4 and 6)

The geometrical shape is shown by using a letter, which in most cases is the first letter of the name of that figure. For example, using letter "t" after the shape indicator

indicates triangle, the letter "r" after the shape indicator indicates rectangle and so on. However, some other logic is also used with the shape indicator. For example, one is tempted to say that shape indicator with letter "s" is "square" but it is not true. In shapes we classify "regular" and "irregular" shapes. The indication of the number of sides of the shape with shape indicator means the regular figure, and the letter or letters which can be treated as acronyms for the shapes may be used for "irregular" figure. For example, a four sided figure where all sides are equal may be a "square" or a "rhombus". In indicating a hexagon, it may be a regular hexagon where all five sides are equal or an irregular hexagon where the sides are not equal. For a square, the mathematical symbol used is

See that the number of sides of a square (4) when indicated in numeral form means the shape square.

Similarly, for a regular hexagon, the symbol used is

whereas for an irregular hexagon, the code used is

⠿ ⠶ ⠣

Here the acronym for hexagon is treated as 'hx'. By applying this logic, one can learn most of the mathematical codes. The purpose of this module is to enable the learner to follow a systematic approach in learning Nemeth mathematical Braille codes and not through memorisation. This module teaches the mathematical Braille codes through the following approaches.

1. **Let the learner understand the different types of codes**

   See the following passage:
   I want to distribute <u>1000</u> Braille slates to <u>25</u> schools @ <u>40</u> per school and therefore, the distribution will be shown as <u>1000</u> = <u>25</u> × <u>40.</u>

   In this passage, four types of codes are used. The descriptions made in English indicate the literary codes, there are numbers such as 1000, 25, and 40 which are written by using number sign, @ which is a script and (=) and (×) are mathematical operations. This distinction helps the learner to learn the mathematical code better by applying reasoning.

2. **Let the learner understand that all mathematical codes cannot be interpreted by a single Braille cell.**
   Start listing the mathematical signs, shapes and scripts known to you and you will realise that they are certainly in hundreds. However, the maximum Braille cell configurations that we get are only 63. The question is how to interpret the different codes using the 63 combinations.

   At this stage, try to find out all the sixty three combinations of Braille cell configurations. There are many ways of exploring but the easiest way is to use the reversal or mirror image concept.

Step 1 **:** *Write down the English Braille letter configurations on the left side and write down the mirror images of these cells on the right side.*

English Alphabet	Braille Code	Mirror image
a	● · · · · ·	· ● · · · ·
b	● · ● · · ·	· ● · ● · ·
c	● ● · · · ·	● ● · · · ·
d	● ● · ● · ·	● ● ● · · ·
e	● · · ● · ·	· ● ● · · ·
f	● ● ● · · ·	● ● · ● · ·
g	● ● ● ● · ·	● ● ● ● · ·
h	● · ● ● · ·	· ● ● ● · ·
i	· ● ● · · ·	● · · ● · ·
j	· ● ● ● · ·	● · ● ● · ·
k	● · · · ● ·	· ● · · · ●

l

m

n

o

p

q

r

s

t

u

v

w

x

y

z

Out of the 63 combinations of the Braille cells, 26 combinations have already been used to indicate the open Braille alphabets. Identify the remaining 37 combinations, by going through the mirror images of each letter.

Mirror image of letter (**a**) is dot 4

Dot 4 has not been used so far and therefore, reverse of 'a' becomes the 27ᵗʰ combination. Similarly, reverse of letter 'b' is dots 4 and 5 (   ) which has not appeared so far and therefore, it becomes the 28ᵗʰ combination. However, the reverse of letter 'c' indicates 'c' only and therefore, it doesn't make another combination. Similarly, reverse of letter 'd' is letter 'f' and therefore, we are still at the 28ᵗʰ combination. Similar are the cases with letters (e, f, g, h, i, and j). The reverse of letter 'k' is dots 4 and 6 (   ) and this combination has not been used so far and therefore, it becomes the 29ᵗʰ combination. Find out the 15 combinations which are not repeated.   Therefore, by analysing the mirror images of the alphabets, you get additional 15 combinations, which will make the total combinations so far as 41. Look for the remaining 22 combinations.

Step 2 : *In Nemeth code, the letters **a,b,c,d,e,f,g,h,i,** and **j,** when written in the lower cell of the Braille cells become numbers **1, 2, 3, 4, 5, 6, 7, 8, 9,** and **0** respectively if preceded by the number sign (   ) (dots 3, 4, 5 and 6). The number cells and their mirror images are as follows:*

Number	Braille code	Mirror image
1	· · ● · · ·	· · · ● · ·
2	· · ● · ● ·	· · · ● · ●
3	· · ● ● · ·	· · ● ● · ·
4	· · ● ● · ●	· · ● ● ● ·
5	· · ● · · ●	· · · ● ● ·
6	· · ● ● ● ·	· · ● ● · ●
7	· · ● ● ● ●	· · ● ● ● ●
8	· · ● · ● ●	· · · ● ● ●
9	· · · ● ● ·	· · ● · · ●
0	· · · ● ● ●	· · ● · ● ●

The numbers stand alone as separate combinations, and therefore, another 10 combinations are added to the already identified 41 combinations, thus making the total combinations into 51. Look at the reversals. The reverses of numbers 1 and 2 are additional combinations. Thus, the total combinations arrived so far are 53. Find out the remaining 10 combinations. How to go about?

Step 3 **:** *The dots 3 and 6 combined with other upper cell combinations will provide another nine combinations. They are as follows:*

Braille cell	Mirror image

Therefore, the total combinations obtained so far are 62. By using all six dots of the Braille cell (⠿) one more combination is added, thus making the total Braille dots configurations to 63. The discovery method is applied when braille dot combinations are introduced in this manner. Repeat this exercise and get familiarized with all the combinations.

Description of the Base Codes and the Branch Codes

Out of the 63 combinations of the Braille dots, 36 combinations are used for the alphabets and ten for numbers and therefore, mathematical Braille codes are basically the combinations of the 27 configurations which may stand alone or appear in combination with the remaining 36 configurations. For better understanding, the 27 combinations may be treated as the base codes. As already explained, the shape indicator (⠱) becomes a base code and all codes which emerge as a result of attachment of different Braille cell configurations with this base code indicate different mathematical codes for shapes, operations, etc., which may be called as branch codes. Therefore, the branch code occupies one or more cells and **it should necessarily have the base code in it as the main indicator.** Let us illustrate this with the groups of braille codes. Call the group of

alphabets as 'A'. Let the ten numbers be called 'N'. Call the Math base code group as 'B' which consists of the 27 combinations. The branch code in mathematics may have the following combinations only.

B alone

B_1 B_2... etc.

B_1 N_1

B_1 , N_1 , A_1... etc.

N_1 A_1 may not become a mathematical branch code since there is no base code present in the combination.

With this understanding of the base and branch codes, the descriptions of the base codes may be taken up.

Understanding the base codes

At this stage, you may not be ready for using the codes in mathematical exercise but you will learn the interpretation of each base code and for what purpose they stand for. Though there are several ways of introducing this, the easy way to understand is to go through the base codes in terms of its frequency of usage. In this section, the meaning of the code is described to a large extent and some examples are also cited to develop the logical thinking of the learner. In base codes too, there are many categories. For example, a base code (⠨) dots 4 & 6 indicates decimal point, structural shape modification, greek letter indication, italic types, shaded shape indication, first inner radical indication etc., but elaborate description of every such code may not be necessary for secondary level mathematics. **In this package, all the codes that appear at the secondary level are explained in detail.** After getting mastery over the secondary level codes, **it is suggested that the learners go through the entire Nemeth code book and study other codes. In fact, this package may be useful for understanding the entire gamut of Nemeth codes better.** In this section, descriptions are provided for the base codes, and detailed descriptions of the branch codes and their usage. When more explanations are needed for rules, please refer to the Nemeth code book.

Base Code 1 : Mirror Image of letter (v)

Numeral Sign (⠼)

Numeral sign is indicated by the mirror image of the letter (v) and the dots configuration are dots 3,4,5 and 6. In Nemeth code, the numerals are indicated by the lower Braille cells, that is by dots 2,3,5, and 6 and therefore, there is no confusion between the upper and combination of lower Braille cells which are used for letters. Numeral sign is used before the number when it appears alone.

2 and 7   ⠼⠃   ⠉   and   ⠼⠃   ⠛

In addition to the function as numeral indicator, (⠼) is also used as closing of a simple fraction which will be introduced later.

Base Code 2: Mirror image of the letter (u)

Plus Sign (⠖)

The reverse (u) indicates the plus sign in mathematics. This has other connotations too. For example, in set language, it is used for the sign "union". As union is nothing but the addition of values, it is appropriate to use the sign "plus". While the numbers are added, there is no need to insert numeral sign after the plus sign since, the plus sign indicates that the cell following the plus sign may be a numeral or a letter. As Nemeth code is using upper cell combinations for English letters and the lower cell combinations for numbers (0 to 9), there is no confusion about the Braille codes for alphabets and numbers and therefore, there is no need to use the numeral sign following the basic operation symbols such as plus, minus, multiplication, and division.   The other basic operations will be discussed in the later sections. The illustration for the use of plus sign is as follows:

x + y   ⠭ ⠖ ⠽

2 + 3   ⠼⠃ ⠖ ⠒

2x + 4   ⠼⠃ ⠭ ⠖ ⠲

Please see that the numeral sign appears before the entire operation but after the plus sign, there is no need to insert the numeral sign again.

Base Code 3: Dots 3 and 6
Minus (⠤)

The minus sign in Nemeth code is indicated by dots 3 and 6 which also resemble the visual sign of minus. Using the minus sign between letters and numbers is illustrated as follows:

a - b = ⠀⠀⠀⠀⠀⠀⠀ ,   9 - 5 = ⠀⠀⠀⠀⠀⠀⠀

Though minus is the general code for dots 3 and 6, it is also used for 'hyphen' and long or short dash. Therefore it is contextual. For hyphen, dots 3 and 6 in single braille cell should be used whereas for short dash, hyphen sign in two cells should be used. For long dash more cells need to be used. The dots configurations are as follows:

Hyphen   - ⠤   Short dash - ⠤⠤

Long dash - ⠤⠤⠤

See that the visual closure of the short dash and long dash resemble the corresponding visual signs "___" and "_____".

Base Code 4: Dot 6
Mathematical Comma (⠠)

In the number 1,456, the comma inserted between the numbers indicates the place value and it is called a mathematical comma.

1,456 = ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

Dot 6 is used for single capitilisation indication. For example, (**a**) is written as ⠠ whereas "A" is written as ⠠⠁. In addition to these connotations, dot 6 when used with other mathematical codes have different connotations, which we shall study in the subsequent

sections. A comma used inside the number, for example 1,326 should be regarded as a numeric sign since it appears as a part of the number. However, the comma which separate number, for example, 45, 68, 89 etc, should be treated as punctuation marks. Therefore, use of dot 6 in the mathematical operation is contextual.

Base Code 5: Dot 3
Ellipsis (⠄)

In writing a mathematical expression which has not ended, we use ellipsis. For example, the infinite series 1 + 2 + 3 + ..., has not ended and therefore, the unending part is to be expressed mathematically. For this, dot 3 is used in successive three cells.

Base Code 6: Dots 1,2,3,4,5 and 6
Cancellation (⠿)

Over-writing is possible in visual writing, whereas in Braille, over-writing is not possible and therefore, punching all Braille dots indicates that it is a cancellation sign. The meaning is that the cell should be simply ignored and the code in the subsequent cell should be taken into consideration.   For example, the expression 2 + ⤬ 4 is written as

Punching of all six dots indicate left and right blunt arrow heads too, which are indicated by ⊢ and ⊣ respectively. However, these codes are not common at the secondary level.

Base code 7: Dot 4 - Mirror image of letter (a)
Script Indicator : (⠈)

As mentioned at the introduction, there are scripts such as @, $, etc., which are also used in regular writing and in the mathematics text. Dot 4 indicates the "Script Indicator". Please see that it is not a mathematical code by itself, and it gets meaning only when it is combined with another Braille cell.

For example, dot 4 followed by letter "a"

indicates @ - at the rate of. Please see that @ resembles (a). This logic may be applicable to many scripts. For example, the dollar sign ($) looks like (s) and therefore, the script indicator followed by letter (s) indicates dollar. Similarly, script indicator followed by letter (l) indicate pound sterling, and so on. At the secondary level, the following scripts are commonly found and therefore, the learner needs to know these.

- ¢ cent

- ϶ reverse membership

- ∈ (an element of)

- % percent sign

- £ (Pound Sterling)

- ∗ Asterik

- $ Dollar

- ✔ Check mark

For other higher order signs using the script indicator you can refer to the Nemeth Braille code book.

Base Code 9: Dots 1,2,3,5, and 6

Opening Parenthesis ( )

Parentheses are used in every expression, whether in literary description or in mathematical equation. For example, the expression (apple) uses '(' as the opening parenthesis and ')' as the closing parenthesis. In numbers too – (x+2), parentheses are used. For open parenthesis, dots 1,2,3,5, and 6 are used. Similarly, '[' (square open bracket) and '{' (curly open bracket) are also used as open parentheses. For changing the simple open parenthesis into a square open paranthesis, there is a need to insert dot 4 before the parenthesis sign. Similarly for curly open parenthesis, dots 4 and 6 are inserted before the parenthesis. Therefore, the three types of open parentheses are written as follows:

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(- ⠶   [- ⠨⠶   { - ⠸⠶

Base Code 10: Dots 2,3,4,5 and 6

Closed Parenthesis (⠶)

Closed parenthesis is the opposite of open parenthesis. The square closed parenthesis and the curly closed parenthesis are also written using the same logic by inserting dot 4 and dots 4 and 6 before the respective parenthesis. Therefore, the different types of closed parenthesis are written as follows:

) - ⠶   ] - ⠨⠶   } - ⠸⠶

You can understand the ease of using the open and closed parentheses through the following examples.

(2+3)   ⠷⠃⠖⠉⠾

[x+4]   ⠨⠷⠭⠖⠙⠨⠾

{5x+7}   ⠸⠷⠑⠭⠖⠛⠸⠾

Please see that the numeral sign is not needed after the opening parenthesis since that itself is an indicator.

Base code 8: (⠨)   - Mirror image of letter (k)

Decimal point, Greek letter indicator, Italic type indicator and Structural shape - modification indicator

⠨ Dots 4 and 6 have many connotations. In fact this base code with combination of different cell configurations indicate a large number of mathematical codes. Let us provide examples for various usages of this code. First this indicates a **decimal point**. For example, when we write 1.564, please note that the point (.) is not a period. It is a point

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which separates the whole number and the decimal part of that number. For this purpose, the decimal point is used. Therefore, 1.564 is written as

Dots 4, 6 indicate **structural shape modifiers** too. What do you mean by a structural modifier? See the literary meaning. Suppose, take a triangle, where all the three sides are equal. By making some modifications in the sides of the triangle, we can get a scalene triangle, isosceles triangle and right angle triangle. The logic here is that you indicate the triangle first and then indicate that the triangle is scalene in nature. Therefore, the code for triangle followed by the code for modification will indicate the modified triangle. You will learn more about this in the section dealing with shape indicator.

Dots 4 and 6 followed by the alphabets indicate the **Greek letters**. In mathematics and science textbooks, Greek letters such as alpha, beta, gamma, delta, etc., are used. In most of the cases, the first letter of the Greek letter is indicated by the first letter of the equivalent English alphabet preceded by dots 4 and 6. However, the use of first letters may not be correct in the case of some Greek letters. See how the Greek letters are indicated below:

Alpha	-	α		Gamma	-	γ
Beta	-	β		iota	-	ι
Delta	-	δ		Kapta	-	κ
Epsilon	-	ε		Lambda	-	λ
Phi	-	ϕ		Mu	-	μ

Nu	-	ν	⠩		Tau	-	τ	⠞
Omicron	-	o			Upsilon	-	υ	
Pi	-	π			Xi	-	ξ	
Rho	-	ρ			Psi	-	ψ	
Sigma	-	σ			Zeta	-	ζ	

In addition to these functions, dots 4 & 6 can also be used to indicate italics of a letter. For example small italic letter '*a*' is written as

Base code11 : Dots 4 and 5 - Mirror image of letter (b)
Superscript Indicator (⠒)

In mathematics, subscripts and superscripts are often used. For example, in writing a^2, there should be an indication that the 2 is a superscript. In visual form, it is clear as the position of 2 itself is different from the baseline. In writing this in Braille, there should be an indication that the 2 following 'a' is above the baseline and therefore, there is a need for an indicator. This indicator is indicated by the dots 4 & 5. Please see that the dots 4 & 5 literally appear in the upper portion of the Braille cell. We normally call dots 1,2,4 and 5 as upper cell and dots 2,3,5 & 6 as lower cell. Please see that dots 4 & 5 appear in the upper cell and therefore, denote the superscript indicator. Therefore, a^2 is written in Braille as follows:

As 2 is a number, you may have a doubt whether or not to insert the numeral sign before the number "2" and after the superscript sign. A rule of thumb is that numeral sign need not be used after an indicator as numeral sign is also an indicator and two indicators together are not necessary. You will understand this rule when you start practicing the usage of these codes.

Taking a clue from the literal meaning of superscript, it is the one which is "up" and therefore, the opposite of "up", which is down is called as the "subscript". If dots 4&5 belong to the upper cell and indicate superscript, what may be dots for subscript which are expected to appear in the lower cell? The dots must be 5 & 6. You are right. Dots 5 & 6 indicate the subscript.

Base Code 12 : Dots 5 and 6 - Mirror image of number (2)
Subscript Indicator (⠰)

Like superscript, which is indicated by the reverse "b", that is by dots 4 and 5, the subscript operation is indicated by mirror image of number 2, that is by dots 5 and 6.

X_2 is written as

Note : There is no need to use subscript indicator if subscript is a numeral to the variable.
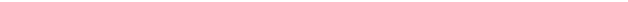
Base Code 13: Dot 5 - Mirror image of the number (1)
Baseline Indicator (⠐)
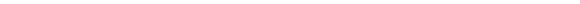
In writing expressions such as $X^2 + Y^2$ etc., the learner should understand that the sign (+) is written at the base level. Though it is clear visually, there needs to be an indication for the tactile learner that the superscript function is over and we are returning to the base before starting the next operation. In the above expressions, X, +, and Y are at the base level, whereas the number "2" is the superscript. Whenever a number is distinguished from the base level by superscript or subscript operations, there is a need to indicate the "returning to the base" by inserting the base line code (dot 5) after the superscript or subscript. Therefore, the above expression is written in Braille form as follows:

Similarly

$x_2 + y_2$ is written as

$x^3 + y_4$ is written as

In writing mathematical expression, dots 5 & 6 are also used for indicating English letters. When English letters are single between operation such as the above expression, there is no need to use the English alphabet indicator. However, it is necessary to use this indicator when more letters appear without preceding and succeeding indicators. For example (ab) should be written as

whereas ab should be written as

Base Code 14 : Dots 1 and 6
Multiplication (*)

In multiplication, more than one sign is used. For example, in the expressions 2 ★ 3 and 2×3, the ★ and × indicate multiplication operation. The symbol ★ is indicated by the dots 1 and 6 which resemble the visual symbol \ . For indicating the × sign, the dots 1 and 6 should be preceded by the dot 4 in the first cell. Therefore, the operations given in this example are written as follows in Braille.

2 ★ 3 =

2 × 3 =

The dot sign (·) (dots 1 and 6) is also used for dot within inclusion (⊂) sign, dot within reverse inclusion (⊃) etc., but these signs are of higher order in nature and do not normally appear at the secondary level.

Base Code 15 : Dots 2 and 6

Division (/)

In division, the signs (/) and (÷) are normally used. For indicating /, dots 3 and 4 are used. Please see that the visual sign resembles the Braille sign too. In literal meaning, multiplication is considered as the opposite of division and in mathematical codes too, the visual presentation for multiplication (dots 1 and 6) and division (dots 3 and 4) look opposite ones. *This logic may be applied wherever possible. For example, dots 4 and 5 indicate superscript since they come on the upper portion of the Braille cell whereas the subscript which is considered to be the opposite of the superscript appears on the lower side of the Braille cell represented by dots 5 and 6. For union in set language, the symbol "reverse u" is used whereas for intersection, the opposite of this symbol - dots 1,4,and 6 are used. Therefore, the 'opposite' logic is applied in most of the cases and applying this logic will help the learner to learn mathematical codes by not memorising but through systematic application of logic.*

Though the symbol (/) is used in normal division, (÷) is also used as a specific sign for division. In order to distinguish this sign from the (/) sign, the Braille symbol for this is written as dots 4 and 6 followed by dots 3 and 4 in the second cell. As dots 4 and 6 indicate the punctuation indicator, addition of this before the division symbol indicates the division sign (÷) The examples using these signs are as follows:

8 ÷ 3   =

4x / 2   =

529 ÷ 7 =

Base Code 16 : Dots 1, 2, and 6

Directly over and inner radical sign

The arrows are written in four forms, viz.,

| ← | - arrow indicating left side | ↑ | - arrow indicating up |
| → | - arrow indicating right side | ↓ | - arrow indicating down |
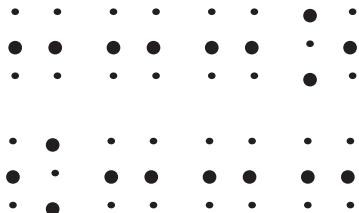
The dots 2 and 5 which are in the middle of the Braille cell are used for the line of the arrow and the arrow heads are used by other characters of the Braille cell. For example, the left arrow head is indicated by reverse "o", that is dots 2,4,and 6 whereas the right arrow head is indicated by the letter "o", that is dots 1,3,and 5. See that they are opposite to each other. Therefore, the right arrow and left arrow are indicated in Braille in the following ways.

In the case of arrows pointing up and down, the shape of the arrow remains the same but the direction of the arrow changes. For indicating down, dots 1, 4, and 6 are inserted in the cell preceding the arrow and for indicating the arrow up, the symbol is preceded by dots 1, 2 and 6. Therefore, the symbol for directly over is dots 1, 2 and 6. This is not only applicable to the arrows but for all applications. For example, in the integral sign $\bar{\int}$, the bar over the integral sign indicates directly over and therefore, it should be written as

Similar application is needed in writing expressions such as $\overline{\lim}$, $\underline{\lim}$, etc. The dots 1, 2, and 6 are also used as inner radical of a radical operation. For example in the operation $\sqrt[3]{5x + 8}$, the $\sqrt{\ }$ is the radical sign and the 3 is in the inner radical. In order to indicate that 3 is in the inner radical, dots 1, 2 and 6 are used before the number, and then the number followed by the radical sign. Therefore, the above operation is written as

Please see that there is no need to use the numeral sign before the number 3 and the insertion of the inner radical sign itself indicates that it is a non-literary operation and the learner will understand that inner radical sign is a part of mathematical code and therefore, the dots 2 and 5 indicate the number 3. As this is an operation that should be completed before proceeding further, the termination indicator needs to be used.

Base code 17 : Dots 4, 5 and 6 - Mirror image of letter (l)

Punctuation Indicator (⠸)

Though dots 4,5 & 6 have many meanings, they basically function as the punctuation indicator. They are also used for indicating boldface letters. For example, we write functions like **a** + b where letter (**a**) is written in bold. Inserting dots 4, 5 & 6 before the letter 'a' indicates that it is bold type whereas letter b is of normal type.

The period at the end of the function is also indicated by

For example, "3." is written as

Dots 4, 5 and 6 are also used for indicating the presence of German letters in the text.

Base code 18 : Dots 1, 4 and 6 - Mirror image of letter (m)

Directly under: (⠹)

In mathematics, certain indicators change the meaning of the functions of the letter, equations, etc. For example, $\underline{\lim}$ with an underscore below the "limit" value is different from the expression $\overline{\lim}$ (over). Similarly $\underline{X}$ and $\overline{X}$ are different. Dots 1, 4, & 6 put before the operation indicates that it is directly under. Again this is an indicator. The mirror image of the letter "m" is represented by dots 1,4, and 6. In writing mathematical expressions, there are occasions when we indicate anything pointing down and this is called "directly under". For example, the arrow where the arrow head pointing downwards is indicated by the directly under sign. Similarly in $\underline{X}$, the _ is called directly under and it is indicated by the directly under sign. The examples are as follows:

<u>lim</u>   -   lower limit

<u>X</u>

∫

Base Code 19 : Dots 1, 2, 4, 5 and 6 - Mirror image of (q)
Termination Indicator (⠹)

As Braille writing in mathematics involves many types of codes such as literary, numerals, symbols, operations, etc., the student should understand that the mathematical operation is over before proceeding with literary codes. For example, every mathematical operation starts with an indicator. In fraction, the fraction opening indicator is placed before the fraction and followed by fraction closing indicator. Similarly, an open parenthesis which precedes the number is always closed by the closed parenthesis at the end of the operation. Therefore, the open and closed operations occur in pairs. However, there are some operations and indicators such as shape and structural modifiers, which have to be closed after the function is over. Therefore, the learner who starts with the indicator to change the meaning of the Braille display should also know that the function is terminated before proceeding further. On such occasions, the termination indicator is used. It is indicated by the reverse "q", which means dots 1,2, 4, 5 and 6.

Scalene Triangle

Base code 20 : Dots 2, 4 and 6 - Mirror image of (o)
Left Arrow Head or Angle

The reverse (o) is indicated by dots 2, 4 and 6 and this is used as the left arrow head of an arrow. In geometry, it indicates the angle. See that the mathematical Braille symbol resembles the visual sign too.

Left arrow head / angle

Base code 21 : Dots 1, 2, 4 and 6 Mirror image of letter "n"

Shape Indicator: (⠫)

This code plays an important role in geometry. For indicating any mathematical shape, dots 1, 2, 4 & 6 should be inserted before the Braille cell indicating the shape. In shapes, there are irregular and regular shapes. By regular, we mean the figure where all sides are equal and by irregular, we mean that at least one side of the figure is unequal with other sides. For indicating the regular figure, we state the number of sides except for triangle. Shape indicator and the letter (t) indicate an equilateral triangle in which case all sides are of same length. For square and figures which have more than 5 sides, the number of sides should follow the shape indicator for indicating the regular shape. For example, shape indicator and number (5) will indicate regular pentagon, shape indicator and number (6) indicates regular hexagon, shape indicator and number (8) indicates regular octagon and so on. For irregular hexagon, shape indicator should be followed by letters (hx), since the single letter (h) is used for indicating a "rhombus". The braille codes for some of the commonly used geometrical shapes are as follows:

Shape	Symbol	Braille
Circle	◯	⠫⠉
Ellipse	⬭	⠫⠑
Parallelogram	▱	⠫⠏
Rhombus	◊	⠫⠙
Is Parallel to	‖	⠿⠇
Right Pointing Arrow	→	⠫⠕⠀⠒⠒⠕
Is Perpendicular to	⊥	⠫⠏

Shape		Braille
Quadrilateral		
Rectangle		
Equilateral Triangle		
Trapezoid		
Angle		
Left Pointing Arrow	←	
Horizontal Two way Arrow	↔	
Square		
Regular Pentagon		
Regular Hexagon		

The mathematical codes as per the Nemeth code are approximately 700. Though all the mathematical codes branch out of these 27 base codes, some base codes have more usage than the other codes. The base codes Script indicator, Punctuation indicator, Shape indicator and Structural shape modification indicator result in most of the mathematical codes, and therefore, mastery over these four base codes and their combinations with other codes becomes vital. Though nearly 700 codes are used in Mathematics, not every code is used at the secondary level mathematics. At this level, approximately 250 codes

are used and the objective of this package is to develop the skills of teachers in these codes.

Different applications using the shape indicator:

The shape indicator is used with two additional operations, namely, inner shape modifier and structural modifier to come up with many other Braille codes.

For example, circle is a simple shape and therefore, it is indicated by

Suppose one wants to put an arrow inside the circle ⊖ , then the inner shape modifier should be used. Here see that arrow is also a shape and inserting it inside the circle will mean a different connotation. *The shape indicator preceded by dots 4, 5, and 6 indicates inner shape modifier.* The Braille configuration of this is as follows:

In this configuration, see the sets of operation. First, you have to put the shape indicator and the letter (c) which indicate that it is circle. Then you are making a change in the circle by inserting the arrow within. It indicates that you are modifying the shape. Therefore, type the inner shape modifier following the "circle". At this stage, the Braille configuration is as follows:

At this stage the learner knows that you are modifying the shape of the circle and the shape to be included inside the circle is an arrow, which again is a shape. Therefore, treat the arrow as an independent shape and insert shape indicator in front of arrow. By doing this, you will get the Braille configuration as follows.

After completing the writing of the entire Braille configuration, you have to close down the operation by inserting the termination indicator at the end. Whenever you use the shape indicator, close the operation by inserting the termination indicator at the end.

Therefore, the Braille configuration for the expression is

⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿

Similar to the insertion of shape inside the circle, assume that we want to insert a symbol (+). Here, the (+) is not a shape, but inserting the (+) inside the circle is modifying the meaning. Therefore, the inner shape modifier should be used but before (+), there is no need to use shape indicator since (+) is not a shape. The Braille configuration is as follows:

⠿⠿⠿⠿⠿⠿⠿⠿

Through the above operations, we understand that the shape can be shown independently, shape shown within another shape, and a sign shown within a shape. Using these operations in exercises will help you remember the procedures as well as codes. One rule of thumb is that you remember the procedures before writing the codes.

Let us illustrate the use of structural modifier with shape indicator. Let us take the shape triangle. It is indicated by

⠿⠿

Though this is the basic sign for the triangle, there needs to be a difference between isosceles triangle, right angle triangle, scalene triangle etc. Here note that you have to change the structure of the shape. Therefore, the structural modifier (dots 4 and 6) should be used. The easiest way to understand this process is that the right angle triangle should be called triangle right angle, isosceles triangle as triangle isosceles and so on so that the child understands that the code for triangle should be written first and then the modifier used to indicate the change of shape. The first letter of the modified shape should normally follow the structural modifier and finally, the operation should be closed by inserting the termination indicator. Therefore, the right angle triangle is written as

⠿⠿⠿⠿⠿ ◺

Similarly, the other types of triangles are written as follows:

Isosceles triangle

Scalene triangle

Like modification of triangle, the other occasion where modification is used is in the case of angles. The shape of the angle is indicated as

Use the same logic used in the case of triangle in modifying the angle and therefore, the modified figures are indicated as follows:
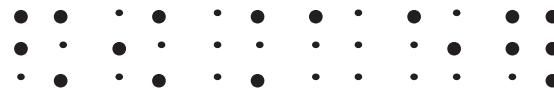
Obtuse angle

Acute angle

Alternate exterior angle

Interior angle

You can see from this section that about 100 mathematical Braille codes are pertaining to the shape indicator, structural modifier and the inner shape modifier. Mastery over this section is important for learning geometry.
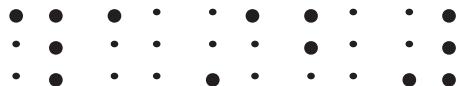
Base Code 22: Dots 1, 4, 5 and 6. Mirror image of (p)
Fraction Opening Indicator

The reverse of (p) is indicated by the dots 1,4,5 and 6. In English Braille contraction, the reverse (p) indicates (th), and in mathematics, it indicates the opening of a simple
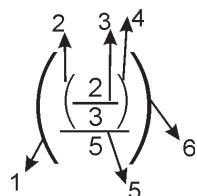
fraction. For example, the child should understand the fraction (a/b) when written within literary codes. In order to distinguish the literary Braille code from the mathematical Braille code, the reverse (p) is used to indicate the opening of the fraction. Similarly, the fraction should also be closed and this is indicated by the inverse image of "reverse p", which is indicated by dots 3,4,5 and 6 which stands for the numeral indicator too. See that the Braille codes have several connotations depending on the context. For example, the number sign when it comes in front of the number is called the numeral sign whereas when it comes at the end of the equation, it indicates the closing sign of the fraction. Therefore, a/b is written as

Please see that this is only a simple fraction. There are complex and hyper-complex fractions too. A complex fraction is a fraction where at least the numerator or the denominator is a simple fraction. For indicating a complex fraction opening, the fraction sign is preceded by dot 6. Similarly, closing of the complex fraction is indicated by the fraction closing sign preceded by dot 6 after the complex fraction is written. The writing of the complex fraction is illustrated as follows:

$$\frac{\frac{2}{3}}{5}$$

See how $\dfrac{\frac{2}{3}}{5}$ is written visually. It is written as

Look at the operations. Operations 2 and 4 are for the first operation which is a simple fraction. The bar marked as 3 is for the simple fraction and therefore, the expression till this stage can be written as

 $-\dfrac{2}{3}$

The bar marked as 5 is pertaining to the complex faction $\dfrac{\frac{2}{3}}{5}$, and therefore, the code for horizontal bar of a complex fraction is ⠒⠒⠒⠒, that is sign of the bar of the simple fraction preceded by dot 6, (⠒⠂) in the preceding cell. Similarly operation marked as 1 is the opening parenthesis of the complex fraction and therefore written as ⠴⠦, that is the fraction open sign preceded by dot 6 in the previous cell. Similarly, the closing of the complex fraction 6 is indicated by ⠴⠴

That is how the operation is written as

Hyper-complex fractions also appear in the text. A hyper-complex fraction is a fraction in which at least the numerator or the denominator is a complex fraction. The example of a hyper-complex fraction is given as follows:
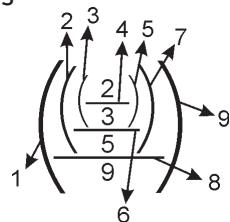
$$\dfrac{\dfrac{\frac{2}{3}}{5}}{9}$$

As the simple fraction opening sign preceded by dot 6 is called complex fraction opening, a complex fraction preceded by dot 6 is called a hyper-complex opening. That means, a simple fraction opening sign preceded by dot 6 in each of the two preceding cells is called a hyper-complex fraction opening. The same logic is applied to hyper-complex closing too. The following example illustrates the written expression of the hyper-complex fraction.
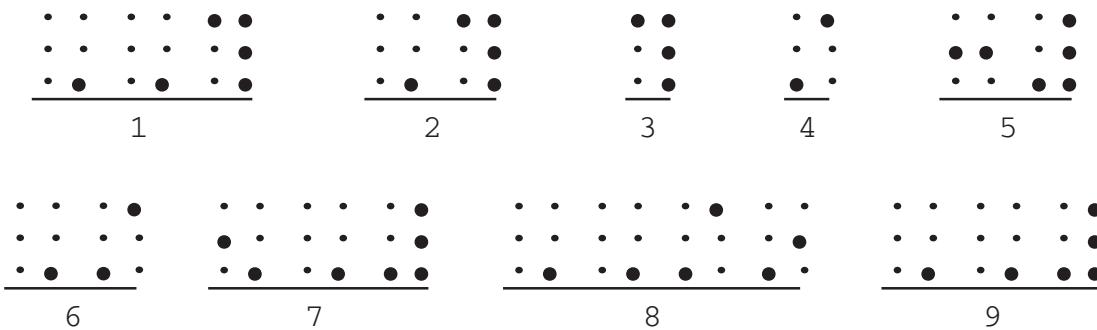
Understand this operation in the visual way too.

See that the operation is written as
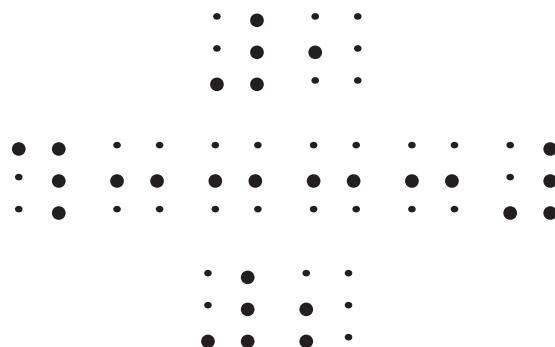
There are nine operations involved in writing this code. Parentheses 3 and 5 pertain to simple fraction, parentheses 2 and 7 pertain to complex fraction and parentheses 1 and 9 pertain to hyper complex fraction. Similarly bar '4' is pertaining to simple fraction, '6' to complex fraction and '8' pertain to hyper complex fraction. The sequence of operation is also in the same order as numbered. Now start writing.

1   2   3   4   5

6   7   8   9

Therefore, the student must be told about the positions of parentheses, the sequence in writing etc., to understand this procedure in a better way.

In writing fractions, there are two ways – the linear way and the spatial way. In linear way, the fraction is written horizontally with the indications for mathematical symbols in between. The examples written so far in this section are using the linear way of presentation. The fractions can be written in the spatial way too resembling the visual presentation. For example, the fraction $\frac{1}{2}$ can be presented in the following way.

Similarly the complex fraction $\dfrac{\frac{1}{2}}{5-x}$ is written as

The written expression for hyper-complex fraction is $\dfrac{\dfrac{1}{2}}{\dfrac{5-x}{9}}$ is

As the entire expression is represented visually, it may be easy for a sighted person to transcribe easily. However, visually impaired children may find the spatial way of presentation difficult as they lose track of the continuity while reading. Therefore, **linear way is the preferred way of presentation**.

While presenting complex and hyper-complex fractions in the linear way, the dividing bars also need to be distinguished. They are represented as follows:

Horizontal bar of a simple fraction

Horizontal bar of a complex fraction

Diagonal bar of a simple fraction

Diagonal bar of a complex fraction

Presentation of various types of fractions indeed is little bit complicated but understanding the procedures will make the learning more effective.

Base Code 23: Dots 1, 5, and 6. Mirror image of (s)
Horizontal Bar

Dots 1,5, and 6 (reverse "s") indicates horizontal bar. In vectors, a bar is placed over the letter to mark it a vector, distinguishing from scalar. For such expressions, the horizontal bar is used after the letter. Example is as follows:

$$\bar{X} \ =$$

Base Code 24: Dots 1, 2, 5 and 6 - Mirror image of (t)
Modular Value

In indicating absolute value of a number, the sign is not taken into consideration. For example, |-3| is 3 since the absolute value is 3. In visual presentation, the vertical line is

marked before and after the number. In Braille, the reverse (t), that is dots 1,2,5 and 6 are placed in the first cell followed by the number and then inserting the dots 1,2,5 and 6 in the third cell indicates the absolute value. In matrices too, this symbol is used. For example,

$\left| 3 \right|$ is written as

$\left| \begin{matrix} 2 & 4 \\ 5 & 6 \end{matrix} \right|$ is written as

Base Code 25: Dots 1, 2, 3, 4 and 6 - Mirror image of (y)
Factorial

In algebra, an expression called factorial is used. It is indicated by the visual symbol (!), which is an exclamatory mark in English braille. While dots 2,3, and 6 are used in literary Braille to indicate the exclamation mark (!), the expression "factorial" in Mathematics (!) is indicated by the reverse (y). 5 !, n !, etc., are written in Braille as follows.

5 ! -   n ! -

Factorial is a concept not very much used at the secondary level mathematics. However, there is a need for the learner to understand the descriptions of all base codes.

Base Code 26: Dots 2, 3, 4 and 6 - Mirror image of (z)
Integral Sign

In Calculus, the expression "integral" is used. For more description of the terminology, refer to the instructional materials. The integral sign is written singularly or in multiple ways. For indicating a single integral sign, reverse (z), that is dots 2,3,4 and 6 are used. For indicating a double integral, two cells are used. Please see the examples below:

$\int$ $\int\int$ $\int\int\int$

The integral function is also not common at the secondary level mathematics.

Base Code 27: Dots 3,4 and 5
Radical Sign $\left(\sqrt{}\right)$

In writing the expression $\sqrt{6y + 9}$, inserting the radical sign before the expression and inserting the termination sign after the expression indicate that the entire operation comes under the radical sign. Please note that there is no need to insert numeral sign after the radical sign as use of radical sign itself will indicate that the expression has been changed from literary expression and therefore, there is no need to use the radical sign and the numeral indicator together. The above expression is written in Braille in the following manner.

You must have understood the logic that should be followed in learning mathematical Braille codes. By applying this logic, you need not memorise the codes. Having understood the way mathematical Braille codes have to be written, you have to do a lot of practice to have a thorough understanding on the usage of these codes. Go through the sections once again should there be further problems understanding these codes. **With this basic understanding, a detailed study of the Nemeth codes from the source book will provide you more clarity.**

Hope you liked the way in which Mathematical braille codes were introduced in this section.

Section 4

Instructional Strategies

INSTRUCTIONAL STRATEGIES

Provision of suitable teaching learning materials for all subjects in general, and mathematics in particular, is a vital ingredient in the education of children with visual impairment. Teaching-learning materials assist the teacher of the visually impaired child in teaching concepts effectively.
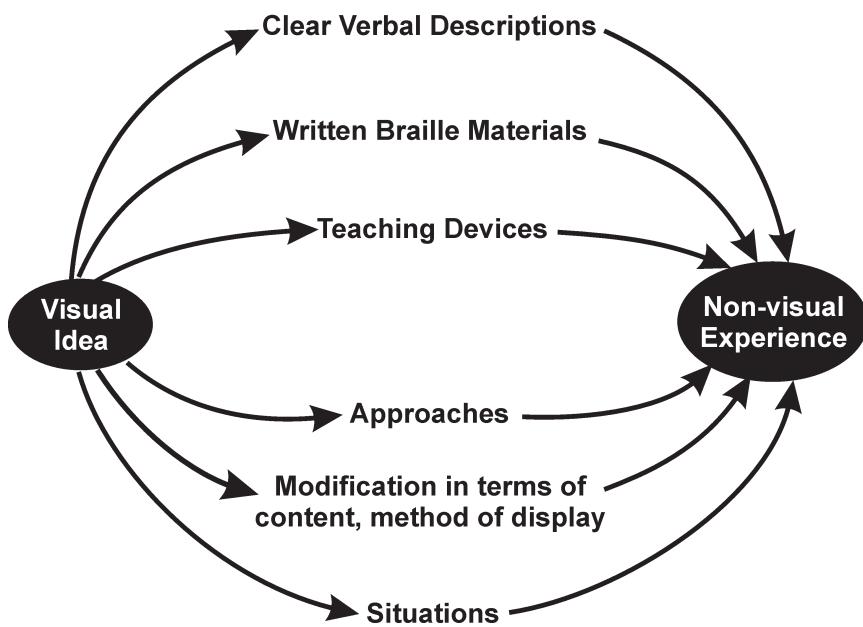
The concepts presented in this section are not only meant for mathematics teachers, but also for the non–mathematics teachers to know mathematical concepts thus enabling them to become better teachers of visually impaired children. The instructional strategies are prepared to help teachers to get an idea of how to go about in handling a child with visual impairment in a mathematics class.

Conversion of visual ideas into non-visual ideas

A child with visual impairment can learn better if a visual idea is presented in the form of a non–visual experience. Any abstract concept when taught to the child supported by relevant non–visual teaching learning material will certainly arouse the interest of the learner and contribute to the learning of the ideas in a better way. Generally a visual idea can be changed into a non-visual experience in six different ways.

Changes in terms of

Clear Verbal Descriptions

Written Braille Materials

Teaching Devices

Visual Idea

Non-visual Experience

Approaches

Modification in terms of content, method of display

Situations

As mentioned in the above illustration, a visual idea can be converted into a non-visual experience in a number of ways. Appropriate teaching learning materials which are tactile in nature, will transform the visual ideas to non-visual experiences which enable the child with visual impairment to understand the concepts effectively. Use of appropriate language in teaching of the concept enables the child with visual impairment to use his auditory sense to learn the concepts. Different approaches in teaching of the mathematical concepts such as part to whole, simple to complex etc., also contribute in the learning process of the child. Further, modification of the content after making necessary adaptations without changing the learning outcomes is a vital factor in changing the visual idea into a non-visual experience thus enabling the child to understand the ideas better. And finally, creation of situational approaches which either exist already or emerged as an outcome of the creativity of the teacher is also a major contributing factor in the learning process of the child.

The approaches mentioned here are not exhaustive. These approaches can be followed separately or they can be clubbed together in any combination to form a different approach to suit the learning needs of children with visual impairment. In the following pages, mathematical ideas that appear in secondary level mathematics have been listed.

For each concept, the mathematical definition followed by adaptation techniques are provided. By going through the instructional procedures described for various concepts in this section, a teacher will become well informed of the secondary level mathematical concepts. It is true that some countries may prescribe more concepts than the ones listed in this document. The possibility of using less concepts may also be true in some other countries. Whether more or less, it is believed that the concepts described in this package are prescribed universally. Concepts may be added or deleted for teaching purposes depending on the country specific curricula for secondary level mathematics.
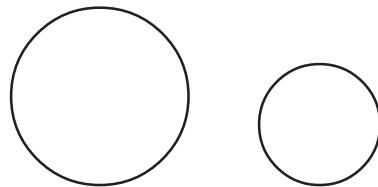
GENERAL CONCEPTS

1. **Big**
    The term big denotes an object which is large in size, amount or extent. The concept may be taught to the child by providing a few balls/objects of varying sizes. Once the child explores all the balls/objects of varying sizes he/she may be able to understand the idea by reasoning and discovery.
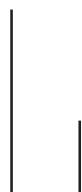
2. **Small**
    The term small denotes any object which is less than the normal size. The idea may be taught by using the same balls/objects of varying sizes which were used to teach the idea of big.

3. **Tall**
    The term tall refers to some object or thing which is more than the average height. To enable the child to understand the idea, allow him/her to touch the shoulders of two pupils, of which one is short and the other is tall. As the child is able to feel by touch that one of them is taller than the other he/she may understand the concept. Wooden sticks of varying heights may also be used to teach the concept.

4. **Short**

    The term short denotes an object or thing which is small in height. The idea may be taught to the child by following the same methodology adopted to teach the concept of tall.

5. **Long**

    Long denotes an object which has a great length in space or time. The wooden strips which were used to teach the idea of tall may be placed horizontally and the child be facilitated to explore their lengths. Now the child needs to be explained the idea of 'long' orally, which he/she may understand by exploration.

    _____

    _____

6. **Greater Than**

    Any object or thing more in number (or quantity or size) is said to be greater than the rest, which are comparatively less in magnitude.

    Two or more number of heaps of beads can be formed and the child be asked to count the number of beads in each heap and the one whose magnitude is more, is greater than the rest of the heaps.

7. **Less Than**

    Any object or thing small in number (or quantity or size) is said to be less than the rest which are comparatively more in magnitude.

    Two or more number of heaps of beads can be formed and the child may be asked to count the number of beads in each heap and the heap whose magnitude is small is the lesser.

8. **Greatest/Biggest/Tallest**

    The term great denotes an object or thing which is considerably above average in amount, extent or strength. The adaptation used to teach the concept of 'big' may also be used here. The idea of 'greatest' among a set of numbers needs to be explained orally, supported by relevant text material in Braille.

9. Least

The term 'least' denotes an object or thing which is smallest in amount, extent, or significance. After explaining the idea orally, the procedure followed in teaching the idea of greatest may be used to teach the idea of least also.

10. Across
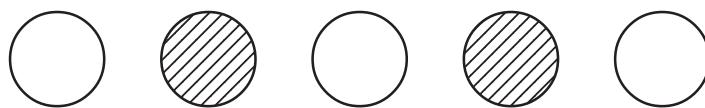
The term 'across' means, from one side to other side of some thing or some object. The idea may be explained orally and then the child be provided a Braille slate. When a particular row of the Braille slate is considered, the cells commence from one side and end at the other side. Exploration of cells on the slate, supported by verbal explanation of the idea may enable the child to understand the concept better.

11. Enlarge

The term 'enlarge' means to make bigger. To enable the child understand the idea, provide him/her with a balloon. When air is blown inside, the balloon enlarges in size which the child can feel by exploration.

12. Alternate

The term 'alternate' literally means leaving a particular number or thing in between and considering the next in order. For instance, 3 and 5 are alternate odd numbers, 4 and 6 are alternate even numbers. The idea may be taught orally supported by relevant text material in Braille.
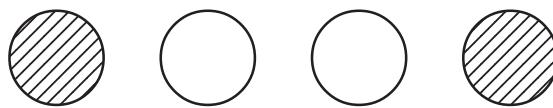
13. Dozen

The term, 'dozen' denotes a group or a set of twelve objects. The concept of dozen may be taught orally supported by relevant text material in Braille.
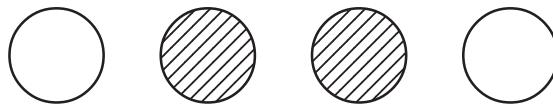
14. Extremes

The word 'extreme' means objects/things at the two ends. The idea may be taught with the help of tactile shapes.

15. Means

The word 'mean' denotes the terms/objects between the extremes. The idea may also be taught with the support of tactile shapes.

16. BC

BC is the abbreviation for Before Christ and is used to indicate a date that is before the Christian era. The idea may be explained orally.

17. AD

AD is the abbreviation for Anno Domini and is used to indicate a date that comes after the traditional date of Christ's birth. The idea may be explained orally.

18. a.m

a.m is the abbreviation for the Latin words *ante meridiem* and is meant for denoting a particular time before noon. The idea may be explained orally.

Note : Concepts 18 to 21 may be taught effectively using a tactile model of a clock.

19. p.m

p.m is the abbreviation for the Latin words *post meridiem* and is meant for denoting a particular time after noon. The idea may be explained orally.

20. Forenoon

Forenoon denotes the time which falls before 12.00 noon. The idea may be taught orally.

21. Afternoon

Afternoon denotes the time which falls after 12.00 noon. The idea may be taught orally. Real life experiences may also be quoted, say for instance the class schedule of the particular student may be considered to explain the idea.

22. Before

The teacher can explain orally so that the child is able to conceptualize the idea of 'before'.

Students may be asked to form a single file so that one student stands behind the other. As each student knows who is standing in front of whom (as it is a familiar group) each student may be asked to name the person who is standing before him/her, thereby helping him/her to understand the idea better.

Also, a few known objects, say, miniature models of different objects / shapes, etc., can be placed one behind the other and each student may be asked to explore and spell out which object / animal is placed before the other.
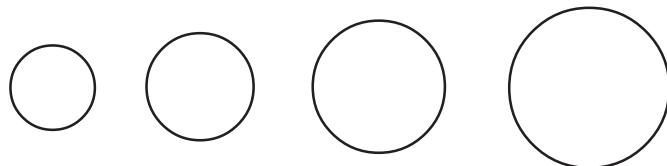
Otherwise, if the child is familiar with the number concept, he/she may be asked to consider a few numbers, say, 2,5,8,3 and be asked whether a particular number, say 5 comes before 3 or 3 comes before 5.

Note : The above procedure can be followed for teaching the concepts of 'after' and 'between' also.

23. Ascending Order

Any arrangement from smallest to biggest is said to form an ascending order. The word 'ascend' literally means 'to go up', 'climb' or 'rise'.

Students in the class may be asked to stand according to their heights and the child with visual impairment may be asked to explore the difference in heights tactually, by touching their heads. The child may proceed from the shortest person to the tallest person.

The idea may also be taught with the provision of a few circular objects of varying size. Once the idea is taught orally, the child may be asked to rearrange the objects
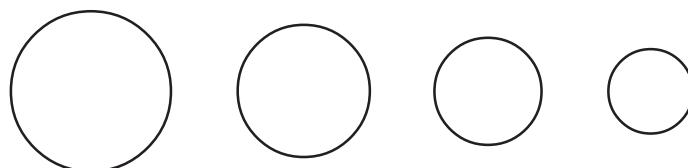
according to their size from smallest to biggest thus enabling him/her to understand the idea of ascending order better.

24. Descending Order

An arrangement from biggest to smallest is said to form a descending order. Descend means 'move down' or 'downwards'.

The procedure using the students themselves, meant for teaching the concept "ascend" can be followed to teach the concept of "descend", the only difference is to start from the tallest and proceed to the smallest.

The procedure followed in teaching the idea of ascending order using the circular objects, may also be followed for teaching the idea of descending order.

25. More

Any object (or thing) whose magnitude is comparatively higher than the rest is said to be more than others.

Few heaps of beads with varying numbers can be formed and the child be asked to explore their magnitude by counting the beads in each heap. The heap whose magnitude is greater is more than the rest.

26. Less

Any object (or thing) whose magnitude is comparatively smaller is said to be less than the rest.

The procedure followed in teaching the concept of 'more' can be followed to teach the concept of less also, with the same heap of beads.

27. Finite / Infinite

Finite denotes any object or thing which is limited in size or extent. Infinite denotes any object or thing which is limitless in space or size. In Set language, finite denotes a set which has countable number of elements and infinite denotes a set which has uncountable number of elements.

Provide the child, first with a handful of beads and ask him to count. Then, provide the child with a handful of sand grains and ask him / her to count the grains. It is evident that the beads can be counted, while the sand grains cannot be counted, thus enabling the child to understand the concept finite and infinite.

28. Predecessor

The word "predecessor" literally means a thing that has been followed by another. In number system, the number which occurs immediately before another is said to be the predecessor to the number considered. While teaching the concept of numbers, the concepts of predecessor and successor may also be taught.

To enable the child to understand the concept, ask the students to form a single file one behind the other, and after the explanation of the concept orally, each child may be asked to name his/her predecessor. As all the students in the file are derived from a known group, the child with visual impairment is also familiar with the remaining members of the group and the child will be able to identify his/her predecessor. The concept of successor can also be taught simultaneously.

5 6 7 8 9

In this group of numbers, 5 is said to be the predecessor of 6.

Once the child is clear with the meaning of the term predecessor the idea may be related to number theory.

29. Successor

Successor denotes the next immediate number. For instance, in natural numbers the successor of 5 is 6 and the successor of 105 is 106. As the idea of predecessor is already taught, the child may easily understand the idea of successor.

The procedure followed in teaching of the idea of predecessor may be followed to teach the concept of successor also.

<div align="center">5   6   7   8   **9**</div>

In this group of numbers, 9 is considered to be the successor to 8.

As in the case of the idea of successor, the concept of predecessor may also be linked to number system once the child is thorough with the meaning of the term successor.

30. Multiplier

In calculating the product of any two numbers, the number by which a given number is to be multiplied is called as the multiplier.

For example, in the multiplication 47 x 68, 47 is called multiplier.

31. Multiplicand

The number which has to be multiplied by the multiplier is called as the multiplicand.

In the example 47 x 68, 68 is called the multiplicand.

32. Dividend

A number to be divided by another number is called the dividend. In the problem $\frac{483}{22}$, 483 is called the dividend.

The child may be taught the ideas of dividend, divisor, quotient and the remainder by demonstrating a division problem in the abacus.

33. Divisor

A number by which another number is to be divided is called the divisor. The idea may be explained orally supported by relevant text material in Braille. In the number $\frac{483}{22}$, 22 is called the divisor.

34. Quotient

The result obtained by dividing one number by the other is called the quotient. The idea may be explained orally supported by relevant text material in Braille.

35. Remainder

The number which is left over when one number does not exactly divide into another is termed as the remainder. The idea may be explained through calculation of the above problem.
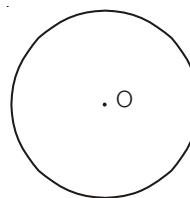
36. Area

The measure of a region or the amount of the surface which any object occupies is called the area of that object.

The child may be asked to keep his Braille slate atop of a table and he/she be asked to explore the top surface of the table and the space occupied by the slate. Now the child may be explained that the space occupied by the slate is its area.

37. Perimeter

$\bar{x}$

The continuous line forming the boundary of a closed figure is called the perimeter. In the case of a square, perimeter equals four times its side, and for the rectangle perimeter equals twice the sum of its length and the width. In the case of a circle, the perimeter is called the circumference and can be calculated by using the formula '$2\pi r$', where 'r' is the radius of the circle.

The idea of perimeter may be explained orally first. Then the child may be exposed to the practical method of finding the perimeter of polygons such as the square and rectangle by actually measuring with thread and compare it by using formulae.

38. Geometry

Geometry is a branch of mathematics which deals with the **study of properties of figures and shapes** and the relationship between them.

39. Point

A point is a small mark having **position but no magnitude/dimension**. A mark made by the tip of the pen may be explained to understand the concept of point. The Braille dot may also be used us a general example, but Braille dot can have a small area too.

40. Ray

A ray starts from a fixed point and extends endlessly in one direction. A ray XY is denoted as $\overrightarrow{xy}$

The idea may be explained orally with the help of an embossed ray prepared on a sheet. The arrow indicates infiniteness.

41. Line

A line is a long narrow mark, which is nothing but a collection of points. A line is straight and extends endlessly in both directions. If the line is not straight, then the collection of points forms a curve.

The idea may be explained with an embossed line prepared on a braille sheet.

42. Line segment

A line segment is a portion of a line with two end points. If P and Q denote the two end points of a line segment, then it is denoted as PQ.

The idea may be explained with the assistance of necessary text material in Braille. An embossed line segment prepared on a sheet may be given to enhance the understanding level of the child.

43. Plane

A plane is a flat surface which extends endlessly in all directions. The concept may be explained to the child orally.
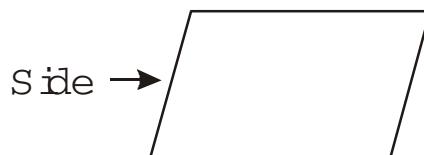
44. Locus

Locus is normally associated with a moving point. The locus of a moving point is the path traced out by it when it moves according to a specified geometrical condition. Note that every point on the locus must satisfy the given condition.

45. Side

A side is a line forming the boundary of a plane figure. While explaining the concept of any geometrical figure, the idea of side may also be explained. A circle has no sides because it is a closed curve and hence its boundary is termed as circumference.
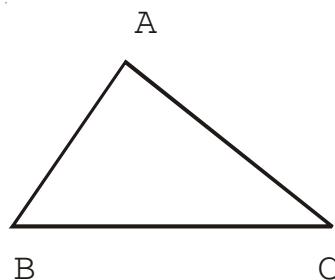
The idea may be explained with the provision of different geometrical shapes like square, rectangle, rhombus, etc., made of hard board for the child to explore and understand.

46. Vertex

The **point of intersection of any two sides of a geometrical figure** is the vertex. The concept of a vertex may be explained while introducing the concepts of square, rectangle, triangle etc.
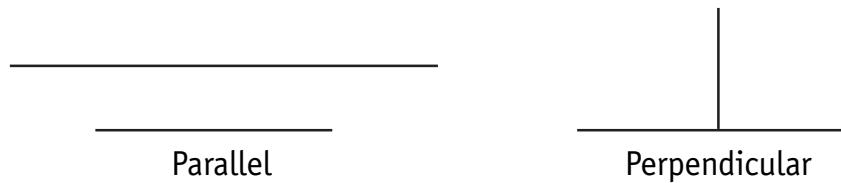
Here A, B and C are the vertices of the triangle $\triangle ABC$.

47. Horizontal / Vertical

Horizontal denotes any object which is parallel to the ground. Vertical denotes an object which is at right angle to the horizontal plane.
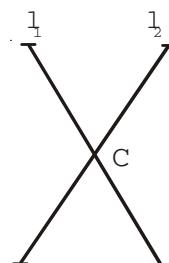
To teach the concept to the child, provide him with a sheet of paper in which two embossed lines meet at right angle and two others are parallel to each other. The embossed lines show that one line is perpendicular to the ground and the other is parallel to it. Related concepts are parallelism, perpendicularity, right angle etc.,

<div style="display:flex; justify-content:space-between;">

———————————————

——————————

Parallel

</div>

Perpendicular

48. Intersection

If two lines meet at a point then they are said to be intersecting lines, and the point at which they meet is called the point of intersection.

A sheet of paper may be given to the child. Ask the child to fold the paper to form a crease. Ask the child to fold the paper once again to form another crease so as to intersect the former at a point. Now explain to the child that the two lines are intersecting as they meet at a point, and the meeting point is called as the point of intersection.

$$l_1 \qquad l_2$$

$$C$$
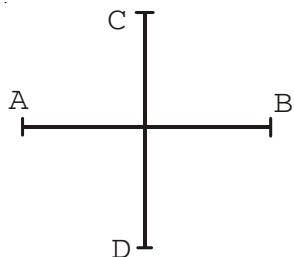
49. Bisection

When the point of intersection divides the two line segments exactly into two halves then the lines are said to bisect each other and the point at which they meet is called as the point of bisection. A sheet of paper in which two line segments, one horizontal and the other vertical, which divide the given paper into halves on either way can be prepared and given to the child. By exploration, the child will be able to understand

that the meeting point of the line segments divides each other exactly into two halves and thus the meeting point is a point of bisection for both the line segments.
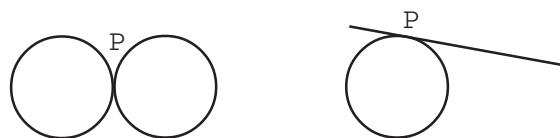
50. Equidistance

If the distance between a particular point to a set of points is equal, then the points are said to be equidistant from the point considered. Provide the child with an embossed circle wherein the distance from the centre to any point on the circumference is always equal. Let the child be helped to measure the distance from the centre to any point on the circumference and on measuring the child will be able to understand that in all the cases the distance is equal and hence any point on the circumference of a circle is always equidistant from the centre.

51. Point of contact

When two circles touch either externally or internally the point at which they touch each other is called the point of contact. The point at which a circle and a tangent touch each other is also called as the point of contact.
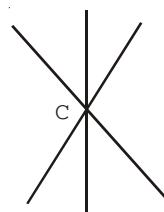
The idea may be taught with the help of either two circular rings which touch each other or a ring and a wooden strip placed suitably touching at a point may enable the child to feel the point of contact and understand.

52. Concurrent lines

Three or more lines passing through the same point are said to be concurrent lines and the point at which they meet each other is called the point of concurrence.

A sheet of paper may be given to the child. Ask the child to fold the paper to form a crease. Ask the child to fold the paper once again to form another crease so as to intersect the former at a point. Let another line also be created by folding the paper. Now explain to the child that the three lines are concurrent to each other as they meet at a point and the point of intersection is called as the point of concurrence.

53. Collinear points

Three or more points lying on the same straight line are said to be collinear points. If the points do not lie on the same straight line, then they are said to be non-collinear points.

Provide the child with a sheet of paper in which three embossed points lie on the same straight line. Ask the child to fold the paper in such a way that the crease formed passes through the three points. Since a straight line can be formed passing through the three points, the given points are said to be collinear. If a straight line cannot be formed using the three points then the points are non-collinear.

A, B & C are collinear points and D, E & F are non-collinear points.

54. Parallel lines

Two non-intersecting lines on a plane which have the same distance between them in all the corresponding points are said to be parallel lines. In other words, two lines are said to be parallel to each other if the distance between them is equal at all points. The child might be asked to explore the edges of a table in which the distance between the edges is equal at all the points.

Also, a tactile diagram in which the distance between any two points of the lines is always same be prepared and given to the child to explore.

Eg. : Railway track is a fine example for parallel lines.
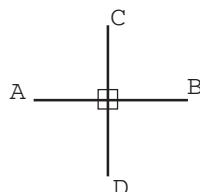
55. Perpendicular lines
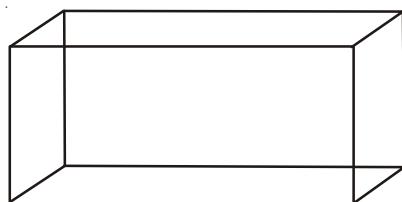
If two lines intersect at right angles to each other then they are called perpendicular lines. In other words, if the angle formed between any two lines is equal to 90°, then the two lines are called as perpendicular lines.

A sheet of paper may be folded twice both horizontally and vertically to form two halves either way. The creases thus formed will intersect at right angles to each other.

56. Skew lines

Lines which lie on different planes and do not meet are called as skew lines. Let the child be given a box. The lines on the top of the box (which is a plane) are different from those lines on the bottom of the box. That is, they are on different planes which do not meet. These lines are called skew lines.

Lines on plane 1 and lines on plane 2 which never meet but parallel are skew lines.

NUMBERS

1. **Natural numbers**

   The numbers 1, 2, 3,... are called as natural numbers. Natural numbers start with 1 and are increased by 1 to get the next consecutive natural number. The set of natural numbers is denoted by the letter N. 1 is the smallest natural number and there is no largest natural number.

   Therefore, N = {1, 2, 3,...}

   The child may be given some beads, say around 20, and he/she may be asked to count them one by one. This activity can be carried out if the child has already been exposed to the basic number system. If the child is to be taught afresh then the idea could be taught stating clearly the basic ideas of the number system before hand.

2. **Whole numbers**

   If the number 0 is included to the set of natural numbers N, then the new set formed is called as the set of whole numbers denoted as W. Note that 0 is the smallest whole number and there is no largest whole number.

   Therefore,   W = {0, 1, 2, 3,...}

   As the child is familiar with the concept of natural numbers already, the idea of whole numbers may be explained with ease.

3. Even numbers

Numbers which are divisible by 2 without leaving any remainder are called as even numbers.

Therefore, 2, 4, 6, 8, 10, 12,... are all even numbers.

To enable the child to know whether a particular number, say for instance, 12 is even, provide the child with 12 beads. Ask the child to group the given beads into two's. If the grouping in two's is possible without a remainder, then the number 12 is an even number.

4. Odd numbers

Numbers which when divided by 2, leaving a remainder 1 are called as odd numbers.

Therefore, 1, 3, 5, 7, 9, 11,... are all odd numbers.

To enable the child to understand whether a particular number is odd, follow the same procedure in identifying an even number. In the process, if a single bead is left over at the end, then the number considered is odd.

5. Prime Number

A number that can be divided by 1 and itself is said to be a prime number. In other words, a number which has got only two divisors is called a prime number.

To enable the child to check whether a number, say 11, is prime or not, provide the child with 11 beads. Ask the child to regroup the beads in equal magnitudes barring the groups of 1's and 11's. If the grouping can be done then 11 is not a prime number. If the grouping in equal magnitudes cannot be done then 11 is a prime number.

Note that 1 is neither a prime number nor a composite number. Also, 2 is the only even prime number.

6. Twin primes

Twin primes are pairs of prime numbers which differ by 2. For example, 3 and 5, 5 and 7, 11 and 13, etc, are all twin primes. The idea may be taught orally supported by relevant text material in Braille.

7. Prime factor

A factor that cannot be expressed as a product of 2 numbers aside from one and itself is called a prime factor. In otherwords, a number that cannot be factorized further is called a prime factor.

8. Composite number

Numbers which are not primes are said to be composite numbers. In other words, a number which has more than two divisors is called a composite number. The procedure followed in the identification of a prime number can be followed for identification of a composite number also.

To enable the child to understand the concept, provide him/her with some beads, say for instance 8 beads. Now, ask the child to regroup the beads other than the groupings of 1's and 8's. If such a grouping could be done then 8 is a composite number.
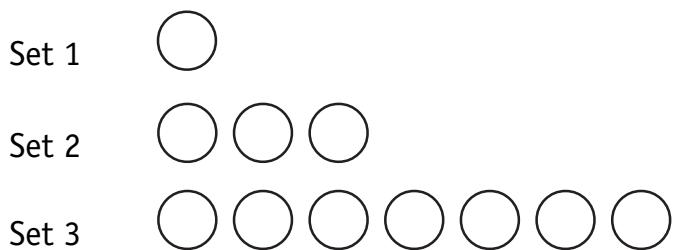
In this case, the beads can be grouped in 2's, 4's and hence a grouping other than the groupings of 1's and 8's is possible and therefore 8 is a composite number.

9. Cardinal Number

The number of elements (objects) present in a set (or group) is the cardinal number of the set (or group).

Few beads might be given to the child and he/she be asked to count them. The number of beads may be increased or decreased and be given to different children, so that the children are able to say different numbers.

Set 1 ◯

Set 2 ◯ ◯ ◯

Set 3 ◯ ◯ ◯ ◯ ◯ ◯ ◯

Here cardinal number of set 1 is 1
Cardinal number of set 2 is 3
Cardinal number of set 3 is 7
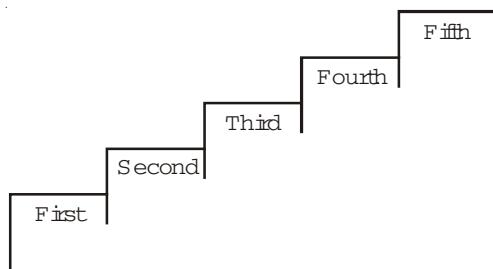
10. Ordinal Number

A number defining an object's position in a series such as first or second is the ordinal number.

The idea may be explained by citing the marks scored by students of the class in a particular test/examination. The student who scored the highest mark obtained the first rank followed by second, third, etc. Few beads may be placed in a uniform pattern and the child be asked to collect them one by one by spelling out the ordinal number of each like first, second, third, etc.

In this concept, the order of the child in the family could also be used.

11. Digit

The numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits. It is by using these ten digits that all numbers are generated in the set of whole numbers.

For example, the numeral for one thousand two hundred and thirty four is written as 1234. Here the digits together are used with place value.

The idea of digits may be explained to the child orally, as by this time he / she must have acquired some basic knowledge regarding natural numbers and whole numbers.

Note : *The system using the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is called as the* ***decimal system or the denary system****. The system of using only two digits namely 0 and 1 is called as* ***binary system****. Also, the system of using 8 digits, namely, 0,1,2,3,4,5,6,7 is called as octal system and the system of using 16 digits, namely, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F is called as* ***hexadecimal system****.*

12. Consecutive numbers

The word 'consecutive' literally means '**following in unbroken sequence**'. In a sequence of numbers, consecutive number means the immediate next number.

A sequence of numbers or shapes may be given to the child and be asked to explore the consecutive of a particular number.

For instance, if the child is familiar with the number concept, explain the idea orally by giving suitable examples such as 7 and 8 are two consecutive natural numbers, 8 and 10 are two consecutive even numbers, 3 and 5 are two consecutive odd numbers, 11 and 13 are two consecutive prime numbers etc.,

13. Face value

Face value denotes the actual value of a digit. The face value of a digit will always be the same as the digit. In the number 789 the face value of the digit 7 is 7, the face value of 8 is 8 and the face value of 9 is 9. Therefore face value of the digits in a particular number is the actual value of the digits. However, depending upon the place they occupy in the number, each digit has a distinct value.

14. Place value

Place value of a digit is the value of the digit in relation to its occupied position in a particular number. The place value of a digit changes according to its position in the number. The place value increases in powers of 10, starting from units. The digits in numbers from right to left have the place values of 1, 10, 100, 1000, 10000 etc. For example, the place value of each digit in 4321 is as follows:

The place value of 1 is 1 ones	1
The place value of 2 is 2 tens	20
The place value of 3 is 3 hundreds	300
The place value of 4 is 4 thousands	4000

Hence the number can be read as four thousand, three hundred and twenty one.

The idea of place value can also be explained to the child orally, emphasizing the difference between the face value and the place value of a particular digit.

Note : Abacus can be used to teach the concept of place value of numbers.

15. Expanded form

The form of writing a number broken into different additive values using the place values is called as expanded form. For instance, the expanded form of the number 6789 is,

$$6789 = 6000 + 700 + 80 + 9$$

$$= (6 \times 1000) + (7 \times 100) + (8 \times 10) + (9 \times 1)$$

Detailed explanation of the idea supported by relevant text material in Braille may be provided to enable the child to understand the concept of expanded form of a number.

16. Multiple

The term 'multiple' denotes a number that may be divided by another, a certain number of times **without leaving a remainder**.

To enable the child to understand the concept, provide him with a heap of beads and ask him/her to study the multiples of 2. That is, first take two beads and then add in 2's consecutively, say up to 50. Once the child is clear with the multiples of 2, he/she can be given another number so as to find its multiples on his/her own. Child should be told that the multiplication table is based on the concept of multiple values.

17. Least Common Multiple

The smallest among the common multiples of two numbers is called their least common multiple.

To find the L.C.M

Step 1.   Write the multiples of the first number

2.   Write the multiples of the second number

3.   Write the common multiples

4.   Write the least common multiple

Eg. : *L.C.M of 3 and 5 is 15 since 15 is the common multiple, least in magnitude for both. The pre-requisite for knowing the concept of L.C.M. is the mastery of the multiplication tables.*

Beads can be used to teach this concept. Considering the above said example, arrange different heaps of beads in multiples of 3 and 5. It is clear that, of the multiples of 3 which are 6, 9, 12, 15, 18, etc, and 5, 10, 15, 20 etc, 15 is the least in magnitude and hence it is the least common multiple of 3 and 5.
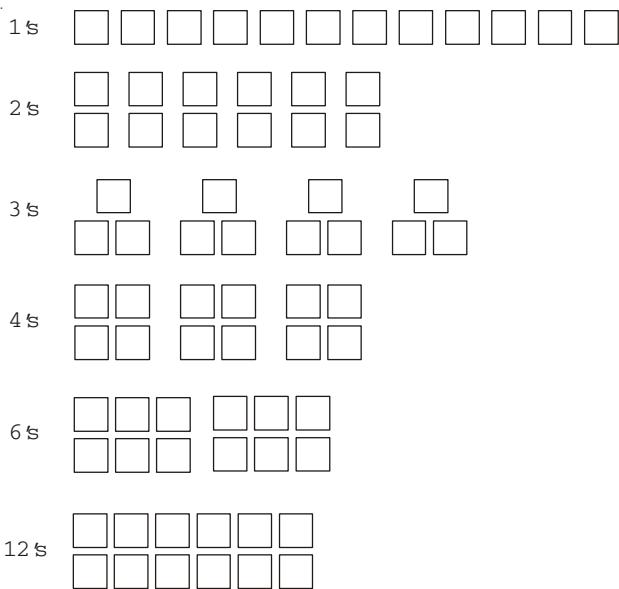
18. Factor

A factor is a number or quantity that when divided with another produces the given number or expression **without any remainder**.

For example, consider the number 12. Note that number 12 is divisible by 1, 2, 3, 4, 6 and 12. Hence the divisors are 1, 2, 3, 4, 6 and 12 and the factors are obtained by excluding the number 1 and 12. Therefore the factors of 12 are 2, 3, 4 and 6.

To teach the concept provide the child with 12 beads. Ask the child to regroup the 12 beads, say, in 1's, 2's, 3's etc., so that each group contains equal number of beads. The number of beads in each grouping becomes a divisor of 12. Once the child has obtained all the divisors, then the idea of factors may be explained orally, that is by saying that if 1 and the given number are excluded then the remaining numbers are the factors.

1 s ☐☐☐☐☐☐☐☐☐☐☐☐

2 s ☐☐☐☐☐
☐☐☐☐☐

3 s ☐ ☐ ☐ ☐
☐☐ ☐☐ ☐☐ ☐☐

4 s ☐☐ ☐☐ ☐☐
☐☐ ☐☐ ☐☐

6 s ☐☐☐ ☐☐☐
☐☐☐ ☐☐☐

12 s ☐☐☐☐☐☐
☐☐☐☐☐☐

19. Perfect numbers

A perfect number is **one that equals the sum of its proper divisors** (except the number itself as a divisor). Note that the proper divisors include all the divisors of the number considered, excepting the given number.

For example,   **6**   =   3+2+1

                28   =   14+7+4+2+1

Observe that in both the cases mentioned above the sum of all the divisors of the number considered equals the given number. Hence such numbers are called perfect numbers.

20. Greatest Common Divisor

Two numbers may have several common divisors. The greatest of all the common divisors is called the Greatest Common Divisor (G.C.D) of the numbers. G.C.D is also known as the Highest Common Factor (H.C.F).

To find out the G.C.D. of 12 and 16, list the divisors of 12, list the divisors of 16 and then select the greatest common divisor

        Divisors of 12   :   1, 2, 3, 4, 6, 12
        Divisors of 16   :   1, 2, 4, 8, 16
        Common divisors   :   1, 2, 4

Therefore, the greatest common divisor of 12 and 16 = 4

Note : 1. **The product of the G.C.D and L.C.M of two numbers is equal to the product of the two numbers**.

2. **The G.C.D of two prime numbers is 1.**

3. **The L.C.M of two prime numbers is their product.**

The idea of finding the G.C.D of numbers can be explained to the child orally. In addition, the pre-requisite for effective understanding of the idea is the mastery of the mathematical tables. If the child is not comfortable with mathematical tables, the concept can be taught through the beads also.

Say for instance, to find the G.C.D of 15 and 18 ask the child to list all the divisors of 15 and the divisors of 18 separately.
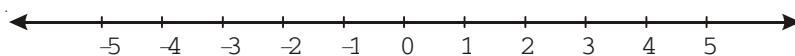
The divisors of 15 are 1, 3, 5, 15
The divisors of 18 are 1, 2, 3, 6, 9, 18

As the divisors of 15 and 18 are grouped in the form of beads separately, on exploration the child will be able to list the common divisors, that is, 1 and 3. Now, among the common divisors, the greatest, which is 3, is the G.C.D.

21. Number line

Number line is a graphical representation of the integers, wherein zero takes the middle place, positive numbers to the right of zero and negative numbers to the left of zero. In the right, the value of a number increases when moving away from zero, and in the left the value of a number decreases while moving away from zero.

Numbers with '+' sign are called positive numbers and numbers with '–' sign are called negative numbers.

Note that since zero corresponds to the origin it does not involve any direction and hence zero is neither negative nor positive.

The idea of a number line is to be taught to the child orally along with the provision of a tactile number line prepared with a thick sheet of paper. As the child explores the number line, he/she will be able to understand the nature of the different numbers both to the right and left of zero in the number line.

22. Integers

Positive numbers, negative numbers and zero together form the set of integers. The set of integers is denoted as Z.

Hence,   Z = { ..., -3, -2, -1, 0, +1, +2, + 3, ...}

In the set of integers +1, +2, +3,...are called as positive integers and -1, -2, -3, ... are called as negative integers. Note that zero is neither positive nor negative.

Note : Positive numbers can be written without the ' + ' sign also.

The idea could be taught to the child with the assistance of a tactile number line.

Related ideas such as the value of the digits may also be explained to the child orally. Enable the child to understand that on the positive side, farther the digit is from zero, greater is its value and on the negative side the closer the digit to zero, greater is its value. Also, the ideas of 'greater than' and 'less than' may also be taught to the child simultaneously with the same tactile number line.

23. Addition of Integers

Integers are compatible for addition. The sum of two positive integers is a positive integer and the sum of two negative integers is a negative integer. Also, if a negative integer and a positive integer are added then the resulting integer will take the sign of the integer which is greater in magnitude.

To add a positive integer with a negative integer, subtract the smaller integer from the greater integer without considering the sign and to the result affix the sign of the greater integer.

In short, when two integers with like signs are added the result takes the same sign and when two integers with unlike signs are added then the result takes the sign of the integer with greater magnitude.

24. Subtraction of integers

Integers are compatible for subtraction also. In subtraction the smaller integer is to be subtracted from the greater integer irrespective of the sign and the resultant must have the sign of the greater integer.

Subtraction of integers may also be taught orally assisted with necessary Braille text material.

25. Reciprocal

For any number other than zero, its reciprocal is obtained by interchanging the positions of its numerator and the denominator.

Note that if the denominator of a number is not given then it is understood that the denominator is equal to 1. Any number with its denominator as 1 is not going to change its value even if it is not written and hence writing the denominator if it is 1 is not mandatory.

For a number $\frac{a}{b}$ its reciprocal is $\frac{b}{a}$. The idea could be explained to the child orally with the typed Braille text.

26. Additive inverse

While adding two integers **if the resultant is equal to zero**, then the two integers are said to be the additive inverses of each other.

Note that the additive inverse of an integer is nothing but the same integer with the opposite sign. That is, if 4 is the given integer then its additive inverse is -4.

The idea of additive inverse can be taught to the child orally. To enhance the understanding level of the child, the idea of negative numbers must be explained

earlier. Once the child is comfortable with the idea of a negative number, he/she may understand the idea of additive inverse easily.

27. Multiplication of integers

Two integers can be multiplied irrespective of their signs. When two integers with like signs are multiplied then the product will be positive and when two integers with unlike signs are multiplied, then the product will be negative.

The rule regarding multiplication of integers can be summarized as follows:

$\times$	Positive	Negative
Positive	Positive	Negative
Negative	Negative	Positive

The idea needs to be explained with the above table prepared in Braille. Adequate practice is needed to master the basic rules regarding multiplication of integers.

28. Division of integers

As in the case of multiplication, division among integers can also be performed irrespective of their signs. The rule regarding the sign of the quotient is also similar as in the case of sign of the product. That is, when division is performed among two integers with like signs then the quotient is positive, and when division is performed among two integers with different signs then the quotient is negative.

Division among integers with different signs can be summarized as follows:

$\div$	Positive	Negative
Positive	Positive	Negative
Negative	Negative	Positive

The concept of division among integers is to be explained with the above table prepared in Braille.

29. Closure property

Let a and b be any two real numbers. Then, **if a + b is also a real number then we say that a and b satisfy the closure property** with respect to addition. As this idea is a simple rule, it may be explained to the child orally.

30. Commutative property

The property in which the **result is unchanged by altering the order of the quantities** is said to be commutative.

Eg. : 3+5 = 5+3

$$\blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare$$

$$\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare$$

Arrange two groups of beads containing 3 and 5 respectively. Ask the child to add the second group numbering 5 with the first one of 3. The result is 8.

Now, form another two groups wherein the first contains 5 beads and the second 3. Ask the child to add both. Here again the sum is 8. Observe that the result is unchanged by altering the order of quantities.

Note : In general,

$$a + b = b + a$$
$$a \times b = b \times a$$

Note that commutative property holds good only for addition and multiplication and is not true in the cases of subtraction and division.

i.e., a-b and b-a, and $\dfrac{a}{b}$ and $\dfrac{b}{a}$ are not equal.

31. Associative property

Let a, b, c be any three real numbers. If, **(a + b) + c = a + (b + c)**, then we say that the three elements a, b and c satisfy the associative property of addition.

The idea may be explained to the child orally. To enhance the understanding level of the child the procedure followed in teaching the commutative property may be followed with three groups of beads.

32. Additive identity

For any two real numbers a and 0, if a + 0 = 0 + a = a, then we say that **0 is the additive identity**. That is with respect to addition, if 0 is added to any number then the resultant is always the number and hence 0 is said to be the additive identity.

The idea may be explained to the child orally.

33. Multiplicative identity

If the product on multiplying two numbers is equal to 1, then the two numbers are said to be **multiplicative inverses** of each other, and **1 is the multiplicative identity**. For any real number 'a' its multiplicative inverse is denoted as $\frac{1}{a}$. That is when 'a' and $\frac{1}{a}$ are multiplied together then the resultant is equal to 1.

Eg. : The product of 5 and $\frac{1}{5}$ is equal to 1 and hence $\frac{1}{5}$ is the multiplicative inverse of 5 and vice versa.

The idea may be explained to the child orally assisted by necessary text material in Braille.

Module 53

RATIONAL NUMBERS

1. **Rational number**

   Any number which can be expressed in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$ is called as a rational number. The set of rational numbers is denoted by the letter Q.

   Eg. : $\dfrac{3}{5}$, $-\dfrac{5}{8}$

   *Note that the **set of rational numbers comprises of natural numbers, whole numbers, fractions and integers**.*

   The idea of rational numbers needs verbal explanation along with necessary Braille text material.

2. **Irrational number**

   Any number which cannot be **expressed in the form $\dfrac{p}{q}$, where p and q are integers** and $q \neq 0$ is called an irrational number.

   Eg. : $\sqrt{2}$ = 1.414... is an irrational number. Similarly, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ and $\sqrt{10}$ etc., are also irrational numbers.

   The idea of irrational numbers may be explained orally with the assistance of necessary Braille text material.

3. **Real Numbers**

Rational numbers and irrational numbers together form the set of real numbers.

The set of real numbers is denoted by the symbol R.

The idea of real numbers may be explained to the child orally supported by relevant text material in Braille.

4. **Dense property**

Between any two rational numbers, there are **innumerable number of rational numbers,** and this property is called as the dense property of rational numbers.

Module 54

FOUR FUNDAMENTAL OPERATIONS

1. Four fundamental operations

The four fundamental operations on numbers are addition, subtraction, multiplication and division. Addition is the process of combining two or more numbers into one number. Subtraction is the reverse process of addition. In subtraction, from a given number a smaller number is taken away and the remainder is found. Multiplication is repeated addition of the same number. Division is repeated subtraction of the same number.

2. Tips for performing the four fundamental operations

In performing the four fundamental operations, the following tips may be of help in enabling the child to perform the activity with ease.

a. While adding numbers it will be easier if complements of 10 with respect to unit digits are understood.

b. To add 9 with a number, add 10 and subtract 1 in abacus.

To add 99 with a number, add 100 and subtract 1.

To add 999 with a number, add 1000 and subtract 1.

Similarly, to add 97 add 100 and subtract 3. To add 996 with a number, add 1000 and subtract 4.

c. (i) To multiply a number by 5, multiply the number by 10 and then divide the result by 2.

Alternate method:

Even number

Divide the number by 2. Attach 0 to the right of the quotient.

Eg. : To multiply 128 × 5

First divide 128 by 2 i.e., 128 ÷ 2 = 64
Therefore, 128 × 5 = 640

Odd number

Divide the number by 2. Leave the remainder. Attach 5 to the right of the quotient.

Eg. : Multiply 279 × 5

279 ÷ 2 = 139

Therefore, 279 × 5 = 1395

(ii) To multiply a number by 25, multiply the number by 100 and divide it by 4.

Alternate method:

Divide the number by 4. If the remainder is,

0, attach 00 to the quotient   **Eg :** 276 × 25 = 69 00

1, attach 25 to the quotient   **Eg :** 377 × 25 = 94 25

2, attach 50 to the quotient   **Eg :** 458 × 25 = 114 50

3, attach 75 to the quotient   **Eg :** 979 × 25 = 244 75

(iii) To multiply an even number by 15, multiply that number by 10 and with the result add half of the value.

Alternate method:

Divide the number by 2. Add the quotient to the given number. Attach '0' to the right of the sum.

Eg. : Consider 72 × 15

$$72 \div 2 = 36$$

$$72 + 36 = 108$$

Therefore. 72 × 15 = 1080

To multiply an odd number by 15, write that odd number, add half of its predecessor and put 5 as unit digit to that added value.

Alternate method:

Divide the number by 2. Leave the remainder and add the quotient to the given number. Attach 5 to the right of the sum.

Eg : Consider 987 × 15

$$987 \div 2 \quad = 493$$

$$987 + 493 = 1480$$

Therefore, 987 × 15 = 14805

d.   To divide a number by 5, multiply that number by 2 and divide it by 10.
To divide a number by 25, multiply that number by 4 and divide it by 100.

All the above mentioned ideas could be explained to the child orally initially. After the child acquires the basic ideas regarding the four fundamental operations, a few exercises involving addition, subtraction, multiplication and division could be given to the child to perform. As the child is going to perform the basic operations with the aid of the abacus, knowledge of abacus becomes a pre-requisite for effective learning of the four basic operations. The above shortcut methods will also help in augmenting the mental arithmetic abilities of the child.

TESTS OF DIVISIBILITY

Tests of divisibility

If the nature of divisibility among numbers is understood then the divisors can be found easily without performing complete division.

- Numbers ending with 0 are divisible by 10.

- Numbers ending with 0 or 5 are divisible by 5.

- Numbers ending with 0,2,4,6 and 8 are divisible by 2.

- If the number formed by the **last two digits of a given number** is divisible by **4**, then the number will be divisible by 4.

- If the **sum of the digits of a number is divisible by 3**, then the number will be divisible by **3**.

- To check whether a particular number is divisible by 6, follow the **test of divisibility by 2 and then by 3**.

- If the number formed by the **last three digits of a given number is divisible by 8**, then the number will be divisible by **8**.

- If the **sum of the digits of a number is divisible by 9**, then the number is divisible by **9**.

- If the **difference of the sums of the digits in alternate places is zero or 11,** then the number is divisible by **11**.

- To check a particular number is divisible by 12, follow the **tests of divisibility by 4 and then by 3**.

- To check a particular number is divisible by 15, follow the **tests of divisibility by 5 and then by 3**.

The idea may be explained orally supported by relevant text material in Braille. To master all the above mentioned ideas, sufficient practice must be given to the child.

DECIMALS

1. Decimals

Fractions having their denominators as 10, 100, 1000 etc., are called as decimal fractions. In other words, decimals are fractions having their denominators as 10.

$\dfrac{1}{10}$ is read as one – tenth and is represented in decimal fraction as 0.1

$\dfrac{1}{100}$ is read as one – hundredth and is represented in decimal fraction as 0.01

$\dfrac{1}{1000}$ is read as one – thousandth and is represented in decimal fraction as 0.001.

In the use of number system with 10 as the base, the place value of a number increases in powers of 10 from right to left and decreases in powers of 10 from left to right.

The idea of decimals may be explained orally distinguishing the value of digits before the decimal point and the digits after the decimal point. The values of the digits such as units, tens, hundreds, thousands, etc., before the decimal point and one-tenth, one-hundredth, one-thousandth, etc., after the decimal point must be explained clearly to the child.

2. Integral part

The digit or digits which collectively occur before the decimal point in a decimal number is/are termed as the integral part or the whole number part.

The place value of digits as far as the integral part is concerned is from the right - units, tens, hundreds, thousands, ten thousands etc. As far as the decimal part is

concerned the place value of the digits starting from the left is one-tenth, one-hundredth, one-thousandth and so on.

To enable the child to understand the concept, provide the child with an abacus and the purpose of the dots in the abacus could be explained emphasizing their significance in the abacus.

3. **Decimal part**
In a decimal number the digits which appear after the decimal point collectively constitute the decimal part. The idea may be explained orally supported by relevant text material in Braille.

4. **Finite (terminating) decimal**
In the expression of a rational number if the process of division is complete, that is upon dividing and if the remainder is zero then the decimal so formed is a finite decimal.

The idea can be explained to the child through demonstration.

Eg : $\dfrac{5}{8} = 0.625$

Here the process of division is complete after a certain stage and hence the decimal formed is finite or terminating decimal.

5. **Infinite (recurring) decimal (Periodic decimal)**
In expressing a rational number if the process of division is incomplete, that is upon dividing if the remainder is not equal to zero then the decimal so formed is said to be an infinite or recurring decimal.

Eg. : $\dfrac{1}{3} = 0.3333333......$

$\dfrac{1}{7} = 0.142857142857...$

As is evident from the example the process of division is not complete and the process continues indefinitely. Hence such decimals are termed as infinite or recurring decimals.

The idea may be explained to the child through Braille text material.

6. Like decimals

Numbers which have same number of decimal places are called like decimals.

    Eg. : 7.8 and 4.5
           1.234, 3490.567 and 198765.543 etc.,.

The idea may be explained through practice using the columns of the abacus.

7. Unlike decimals

Numbers which do not have same number of decimal places are called unlike decimals.

    Eg. : 3.98, 2.5 and 45.098765 etc.,

The idea may be explained through practice using relevant text material in Braille.

8. Conversion to like decimals

A given set of decimals can be converted to like decimals by attaching sufficient number of zeros to the right of it.

Eg. : Convert the following into like decimals: 1.008, 9.7, 11.7654, 0.123456

Here the maximum decimal place is 6, that is in 0.123456

Therefore,   1.008   = 1.008000
                9.7   = 9.700000
                11.7654 = 11. 765400

The idea may be explained orally assisted by relevant text material in Braille. The abacus is useful to explain this concept as the decimal point placement starts from the dots on the separation bar and the digits are placed on the right side of the dots.

9.   Addition of decimals

To add two or more decimals, arrange the numbers in such a way that the decimal points of the numbers are in the same column and the digits of the same place value are in the same column. Then add the numbers and put the decimal point directly under the decimal points of the numbers.

Eg. : Add : 14.5764 + 2.321 + 104.82

$$
\begin{array}{r}
014.5764 \\
002.3210 \\
104.8200 \\
\hline
\mathbf{121.7194} \\
\hline
\end{array}
$$

If unlike decimals are given, convert them into like decimals and then to be added. When the child is comfortable with normal addition, decimal addition can be performed by him/her with ease. The only area which needs adequate attention is the placing of the decimals in the same column. The child must clearly be explained that, once the decimal point is taken care of then the process is relatively simpler. The teaching of this concept through abacus will facilitate better understanding. *(Refer the section on abacus to know how decimal addition is performed)*.

10.   Subtraction of decimals

To subtract the decimals, arrange the decimals in such a way that the decimal points are in the same column and the digits having the same place value are in the same column. Then subtract them and put the decimal directly under the decimal points of the numbers. If unlike decimals are given, convert them into like decimals and then subtraction has to be carried out.

Eg. : Subtract : 156.4734 – 121.1432

$$
\begin{array}{r}
156.4734 \\
121.1432 \\
\hline
\mathbf{35.3302} \\
\hline
\end{array}
$$

The process of subtraction of decimals is also similar to normal subtraction, the only difference being the arrangement of the decimal points in the same column.

11. Multiplication of decimals

To multiply a decimal by another decimal, first remove the decimal point and perform the ordinary multiplication. In the result, place the decimal point as many places from the right as the total number of decimal places in the numbers.

Initially, enable the child to understand that multiplication of decimals needs removal of the decimal points as the first step. Once the decimal is removed, the process is similar to ordinary multiplication. After multiplication is over, then the decimal point must be placed in the final result based on the number of decimal places in the given numbers. Use of abacus for calculation and writing down the numbers in Braille would enable the child to understand the process better.

Eg : Multiply : 15.21×14.12

$$
\begin{array}{r}
1521 \\
1412 \\
\hline
\mathbf{3042} \\
1521 \\
6084 \\
1521 \\
\hline
\mathbf{2147652} \\
\hline
\end{array}
$$

Therefore $15.21 \times 14.12 = 214.7652$

12. Division of decimals

To divide a decimal by another decimal, first convert the denominator (the divisor) into a whole number by multiplying by 10, 100 or 1000 etc., whichever is suitable. Then multiply the numerator (the dividend) also by the same number and then divide it.

Make the divisor a whole number by multiplying it by suitable power of 10 and then divide.

Eg. :   Divide : $\dfrac{8.2648}{0.002}$

0.002 has 3 decimal places. So multiply it by 1000 to make it a whole number. When the denominator is multiplied by a number the numerator should also be multiplied by the same number.

Therefore, $\dfrac{8.2648}{0.002} \times \dfrac{1000}{1000} = \dfrac{8264.8}{2} = 4132.4$

Decimal division is similar to ordinary division except for the removal of the decimal points by multiplying both the divisor and the dividend by a suitable multiple of 10. Emphasize that both the dividend and the divisor are to be multiplied by the same multiple of 10. Once the decimals are removed then decimal division becomes ordinary division. Use of abacus comes in handy for understanding this process.

13. Ratio

Ratio means comparison of two similar quantities by division. The ratio should be in the lowest form. The order in the ratio is very important and can never be interchanged. Note that ratio has no unit. Usually the symbol " : " is used to denote "ratio".

In the ratio a : b , 'a' is called the precedent and 'b' is called the antecedent. The idea of ratio can be explained using classroom situation.

If the particular class contains both boys and girls, an ideal example may be quoted by taking the boys and the girls into consideration. Say for instance, if the class contains 15 boys and 8 girls, then the ratio of the number of boys to the number girls in the class is in the ratio 15 : 8.

14. Comparison of ratios

Two or more ratios can be compared by converting the given ratios into fractions with the same denominator.

Eg. : Compare 3 : 4 and 4 : 5

Here the L.C.M. of the denominators 4 and 5 is 20.

$$\dfrac{3}{4} = \dfrac{3}{4} \times \dfrac{5}{5} = \dfrac{15}{20} \qquad\qquad \dfrac{4}{5} = \dfrac{4}{5} \times \dfrac{4}{4} = \dfrac{16}{20} \qquad\qquad \dfrac{16}{20} > \dfrac{15}{20}$$

Therefore, $\dfrac{4}{5}$ is greater than $\dfrac{3}{4}$

Hence, 4 : 5 is greater than 3 : 4

The idea may be explained through exercises. To get mastery of the idea, the child may be asked to work out a few problems in ratio on his own.

15. Proportion

Proportion is an equality of two ratios. In any proportion the product of two extremes will always be equal to the product of the two means.

Eg. : The cost of 4 notebooks is 24 and the cost of 7 notebooks is Rs. 42.   What is the proportion?

The ratio of two quantities   =   4 : 7

The ratio of their costs   =   24 : 42

Therefore, the proportion is 4 : 7   =   24 : 42

Means

Extremes

Note *: In the above proportion, the first and the fourth terms (4 and 42) are called the extreme terms or extremes. The second and the third terms (7 and 24) are called the middle terms or means.*

The idea may be explained to the child through demonstration by using beads. Emphasis must also be laid in enabling the child to work out a few sums on his own.

16. Direct variation

If two quantities vary always in the same ratio then they are said to be in direct variation. In other words, if two quantities vary inversely such that one of them increases as the other increases, they are said to be indirect variation.

Eg. 1 :

A pen costs Rs.10. What will be the cost of 5 pens, 8 pens and 10 pens?

Number of pens	1	5	8	10
Cost	10	50	80	100

Note that $\dfrac{1}{10} = \dfrac{5}{50} = \dfrac{8}{80} = \dfrac{10}{100}$

Therefore, the number of pens and their cost are in direct variation.

Eg. 2:

  Let the cost of one candy be 5 units.
  Then, the cost of two candies will be 10 units.
  And the cost of three candies will be 15 units.

From the above example it is clear that as the number of candies increases, the cost also increases and hence in this case the candies and the cost are said to be in direct variation.

The idea may be made simpler by taking a real life experience. Assume that the child is buying some chocolates from a shop. The more the number of chocolates he/she buys, more the money he/she has to pay. Less the number of chocolates he/she buys, less the money he/she has to pay and hence the number of chocolates bought and the money paid are in direct variation.

17. Inverse variation

If two quantities uniformly vary such that when one of them increases the other decreases and vice versa then the quantities are said to be in inverse variation.

Eg.   : Supposing a person wants to walk from a place A to B. Let us assume that for a normal person this journey takes 30 minutes.

Once started, if the person walks faster than a normal person then the time decreases. And if the person walks slower than a normal person then the time will be more than 30 minutes, that is, the time increases. Thus when one quantity increases the other decreases and when one decreases the other increases, thus forming an inverse variation. Here the speed in walking and the time taken are showing inverse variation.

PERCENTAGE

1. Percentage

A percentage is a fraction whose denominator is 100. The symbol for percentage is "%". The word percent is an abbreviation for the **Latin word** "**Percentum**" which means per hundred or hundredths.

To convert a percent into fraction, multiply the given number by $\dfrac{1}{100}$ and write the resulting fraction.

To convert a percent into a decimal remove the % symbol and move the decimal point, two places to the left.

To convert a decimal into a percent, move the decimal point two places to the right and insert % symbol. To convert a fraction into a percent, multiply it by 100%.

The idea of percentage may be explained orally with relevant text material in Braille. The marks of the students in a particular test/examination may be converted to percentage, if the particular test/examination is not for 100 marks. Also, enable the child to understand that percentage of any mark scored out of 100 will remain the same as the mark.

Eg. :

1. Convert 38% into a fraction. $38\% = \dfrac{38}{100} = \dfrac{19}{50}$

2. Convert 16.5% into a decimal. $16.5\% = \dfrac{16.5}{100} = 0.165$

3. Convert 0.125 into a percent. $0.125 = \dfrac{0.125 \times 100}{100} = \dfrac{12.5}{100} = 12.5\%$

4. Convert $\dfrac{3}{4}$ into a percent. $\dfrac{3}{4} = \dfrac{3 \times 100}{4 \times 100} = \dfrac{75}{100} = 75\%$

2. Profit and loss

The money paid by a shopkeeper to buy an article from the manufacturer or wholesale dealer is said to be the **cost price** of the article, which is normally denoted as **C.P.**

The price at which the shopkeeper sells the article to a consumer which will normally be more than the cost price is called the **selling price** of the article and is denoted as **S.P.**

If the selling price of an article is greater than the cost price, then the shopkeeper earns a gain or profit. The word 'profit' literally means a financial gain.

Thus,   gain or profit   = S.P – C.P

Further, C.P   = S.P – Profit

S.P   = C.P + Profit

If the selling price of an article is less than the cost price then the shopkeeper suffers a loss.

Therefore, Loss   =   C.P – S.P

Further,   S.P   =   C.P – Loss

C.P   =   S.P + Loss

Eg. : An article was bought for 100 units and sold for 150 units. Find out the profit or loss.

C.P of the article = 100 units

S.P of the article = 150 units

Here, the S.P is greater than the C.P, and hence there is a profit.

Therefore, profit   = S.P – C.P

                         = 150 – 100

                         = 50 units.

To enable the child with visual impairment to understand the concept, provide with two groups of beads wherein one contains more number of beads than the other. Treating the group with more number of beads as the selling price and the one with less number of beads as the cost price the concept can be taught to the child. Similarly the concept of loss can also be taught with the same group of beads.

Note : Profit Percentage $= \dfrac{\text{Profit}}{\text{C.P}} \times 100\%$
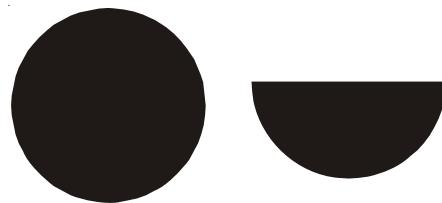
FRACTIONS

1. Whole/Part

The concept of whole/part can be taught to child by providing real life experiences, say for instance, an apple or lemon be given to the child and be asked to explore its shape and size. Later, the object may be cut into pieces, say halves or even of lesser magnitudes and the child be given the opportunity to explore their size. Once the child is given the opportunity to explore the same apple/lemon first as a whole and then as parts, he/she will be able to conceptualize the idea of whole/part.

Otherwise, to make the teaching process still simpler, first provide the child with a full sheet of paper and then tear the paper into parts, say into four parts and provide to the child to explore both, first as a whole and then in parts.

2. One–fourth / One-half / Three- fourth
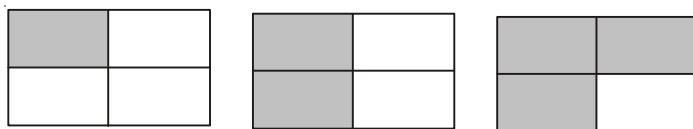
Half denotes either of two equal or matching parts into which something is or can be divided.

To teach the concept to the child, provide him/her with a heap of even number of beads. Ask the child to divide the given heap, into two smaller heaps to contain same number of beads in each.

The concepts of quarter and three fourth can also be taught with the same heap of beads dividing the same into four heaps.

The idea could be taught using a sheet of paper also. A sheet of paper folded vertically and horizontally so as to form 4 equal parts can be used to explain the concepts. The four equal parts of the paper can be torn and be given to the child to explore, or otherwise different textures can also be used to teach the concept.

3. Fraction

A fraction is a part or parts of a whole. In other words, a fraction is a number which is not a whole number.

Provide the child with few beads, say numbering 10. Ask the child to take one out of it. Now the bead in possession is a part of the whole of 10 beads and hence it is a fraction, which is one-tenth of the beads. Similarly, if three beads are taken, then it is also a fraction which is three–tenth of the beads. Ensure that the child does not misunderstand the concept, by taking the whole as less than 10 when some beads are taken away from the whole.

The idea may also be explained by paper folding. Take a square sheet of paper and fold it vertically and horizontally so as to form parts of the whole paper. Now to enable the child to understand the concept, distinguish the required number of parts by a different texture and allow the child to explore. For example, if the paper forms 4 equal parts, and to teach a particular fraction, say $\frac{1}{4}$, distinguish one of the four parts by a different texture and explain the child that the portion taken into consideration is one of the four equal parts and hence it is a fraction, that is $\frac{1}{4}$.

4. Proper fraction

A fraction less than one, that is with the numerator less than the denominator is said to be a proper fraction. In other words, a fraction whose numerator is less than the denominator is called a proper fraction.

The concept of proper fraction may be explained orally and then the child be given a few fractions printed in Braille and may be asked to identify the proper fractions in them.

Eg. : $\dfrac{4}{3}, \dfrac{5}{3}, \dfrac{7}{11}, \dfrac{12}{7}$

Here $\dfrac{7}{11}$ is a proper fraction because the numerator 7 is less than the denominator 11.

Once the child is comfortable with the idea of fraction and proper fraction, the idea of improper fraction can be taught orally supported by relevant text material in Braille.

5. Improper fraction

In a fraction if the numerator is greater than its denominator, then it is called an improper fraction.

For example, $\dfrac{5}{4}, \dfrac{11}{7}, \dfrac{15}{9}$ are improper fractions as the numerator is greater than the denominator in each fraction.

Note : 1. A proper fraction is always less than 1

2. An improper fraction is always is greater than 1

6. Mixed fraction

A number consisting of a natural number and a fraction is called a mixed fraction. For example, $3\dfrac{1}{5}, 17\dfrac{3}{8}, 7\dfrac{5}{11}$ are mixed fractions.

The idea of mixed fraction can be taught to the child orally assisted by necessary written material in Braille. Once the idea is clear to the child he/she may be asked to give examples for mixed fractions, thereby facilitating the child to understand the concept better.

7. Like fractions

Fractions with the same denominator are called as like fractions.

For example, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{13}{5}$ are like fractions.

The idea of like fractions can also be explained to the child orally. Few paper folding in which the whole remains the same and the difference only in terms of the part, that is the numerator, may be provided to the child to explore and the child may be explained that in all fractions formed through the paper, the denominator, that is the whole is same and the parts, that is the numerators are only different. Hence all the fractions formed through the different parts are like fractions since in all the cases the denominators are same.

$\frac{1}{2}\left(\frac{8}{16}\right)$				$\frac{1}{2}\left(\frac{8}{16}\right)$			
$\frac{1}{4}\left(\frac{4}{16}\right)$		$\frac{1}{4}\left(\frac{4}{16}\right)$		$\frac{1}{4}\left(\frac{4}{16}\right)$		$\frac{1}{4}\left(\frac{4}{16}\right)$	
$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$	$\frac{1}{8}\left(\frac{2}{16}\right)$
$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$	$\frac{1}{16}$ $\frac{1}{16}$

8. Unlike fractions

Fractions having different denominators are called unlike fractions.

For example, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{7}$, $\frac{6}{11}$ are unlike fractions.

When the child is through with the idea of like fractions, the concept of unlike fractions can be taught emphasizing the difference between the two that numerators are different in the case of unlike fractions.

As in the case of like fractions, paper folding may also be used to teach the concept of unlike fractions, with different papers folded suitably to form different denominators.

$\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$ are called unlike fractions.

9. Equivalent fractions

When two or more fractions represent the same part of a whole, the fractions are called equivalent fractions. In other words, two or more fractions are said to be

equivalent if after reducing each fraction to its lowest terms, the value of the fractions remains same.

For example, $\dfrac{1}{2} = \dfrac{2}{4} = \dfrac{3}{6} = \dfrac{4}{8}$

Note :

1. *A fraction remains unaltered in value if both the numerator and the denominator are multiplied by the same number.*
2. *A fraction remains unaltered in value if both the numerator and denominator are divided by the same number.*

The idea may be explained to the child through some exercise. Prior knowledge of reducing a fraction to its lowest terms may enable the child to understand the idea better. These concepts may also be taught through the fraction operations in abacus.

10. Comparison of fractions

To compare two or more fractions, convert them into like fractions and then the fraction which has the greater numerator is said to be greater than the rest.

Steps : 1. Find the L.C.M. of the denominators.

2. Convert the fractions with L.C.M. as the common denominator.

3. The fraction having the greater numerator is the greater fraction.

The concept of comparison of fractions may be taught to the child through demonstration using the abacus. Once the child is clear with the idea, he/she may be asked to compare a few fractions on his/her own.

Eg. : Compare $\dfrac{2}{3}$ and $\dfrac{3}{5}$

$$\dfrac{2}{3} \times \dfrac{5}{5} = \dfrac{10}{15}$$

$$\dfrac{3}{5} \times \dfrac{3}{3} = \dfrac{9}{15} \qquad \therefore \dfrac{2}{3} > \dfrac{3}{5}$$

11. Reducing a fraction to its lowest terms

To reduce a fraction to its lowest form, both the numerator and denominator should be either divided by all their factors in succession or divided by their G.C.D. once.

The process of reducing a fraction to its lowest terms is to be explained to the child clearly supported by necessary text material in Braille. To master the idea, the child needs to be given adequate practice in reducing a fraction to its lowest terms on his/her own.

12. Addition, Subtraction, Multiplication and Division of Fractions

For addition, subtraction, multiplication and division of fractions, follow the procedures described in abacus calculations (Modules 6 to 13).

13. Multiplicative inverse

 If the product of two fractions is equal to 1, then each of the fractions is said to be the multiplicative inverse of the other.

Eg. : Consider the fraction $\dfrac{5}{7}$

Note that $\dfrac{5}{7} \times \dfrac{7}{5} = 1$

Therefore, the multiplicative inverse of $\dfrac{5}{7}$ is $\dfrac{7}{5}$ and the multiplicative inverse of $\dfrac{7}{5}$ is $\dfrac{5}{7}$.

MEASURES OF AREA

1. Measures of area

The measure of a plane region or the amount of surface which any object occupies is called the area of that object.

To enable the child to understand the concept of area better, ask the child to keep his Braille slate on the table. On exploration, the child might be able to understand that the slate is occupying a part of the surface of the table. Now the child can be explained that the space occupied by the slate on the table is the area of the slate.

The unit of area must always be denoted as square units. That is if the side of an object is given in terms of meter then the area of the object should be expressed in terms of square meters.

Note : Areas of smaller objects such as the Braille slate, textbook can be expressed as square centimeter (sq.cm) and for areas of larger regions such as tennis court, football ground etc., the unit to be used is square meter (sq.m)

Land area is still larger and hence they are expressed in square meters or hectares.

100 sq. m   =   1 are

100 are   =   1 hectare

10000 sq.m   =   1 hectare

Once the child is clear with the meaning of area, the method of finding the area of different geometrical figures may be explained. While enabling the child to find out the area of a surface, various symbols that are used in the formula must be explained

clearly. After introduction of the idea, sufficient practice must be given to the child to memorize all the formulae relating to area of different geometrical figures. Memorisation is necessary but mere rote memory will not least long. The child must be helped to understand the different parts of the formulae.

2.   Mensuration
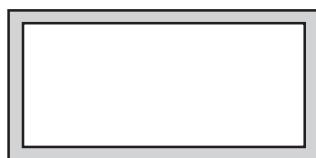
The branch of mathematics which deals with the measure of lengths, angles, areas, perimeters and volumes of plane and solid figures is called as mensuration.

3.   Area of rectangular pathways

In mensuration, finding the area of pathways and finding the area excluding the pathways finds a prominent place. Say for instance, if a garden is in the shape of a rectangle with pathways on all the four sides of the rectangle and if the area of the garden excluding the pathway is to be found out, the formula should be used:

Area of the garden   =   Area of the outer rectangle – Area of the pathway

A cutout in the form of a rectangle with a rectangular pathway inside may be provided to the child to understand the idea and when the child is clear with the idea, he/she may be taught the method of finding the area of the rectangular pathway. The braille cells of the braille slate may also be used to explain this concept.

4.   Area of a square

Area of a square refers to the amount of surface enclosed by the square.

Area of a square =   side $\times$ side

In symbols, A   = a $\times$ a

$\qquad\qquad\quad$ = a^2 square units

Here, the symbol A denotes the area and 'a' denotes the measure of a side.

The idea may be explained to the child orally with the assistance of necessary text material in Braille.

5. Perimeter of a square

The length of the boundary of any closed figure is called its perimeter. In the case of a square, its perimeter equals four times the length of its side.

Thus, perimeter of a square = 4 × measure of the side

In symbols, P = 4 a , where P denotes the perimeter and 'a' denotes the side.

To enable the child to understand the idea, first the child may be explained the concept orally. Once the child is clear with the idea, provide him/her with a square made of wood or plastic along with a piece of thread. Ask the child to measure the lengths of the four sides by placing the thread on the sides of the square starting and ending at the same point. Now ask the child to measure the length of the thread which is the perimeter of the square. Then the child may be asked to fold the thread to form four parts and find out that each part will be equal to the side of the square thus enabling him/her to understand that the perimeter of a square is four times that of its side.

6. Perimeter of a rectangle

Perimeter of a rectangle is obtained by adding the measures of the length and width and then multiplying it by 2.

That is, perimeter   = 2 lengths + 2 widths

                          = 2 (length + width)

In symbols,   P   = 2l + 2b   or   P = 2 (l + b)

The procedure followed in finding the perimeter of a square may be followed in enabling the child to understand the idea of the perimeter of a rectangle also. After finding the perimeter of the rectangle, enable the child to conceptualize that its perimeter is equal to twice the sum of its length and the width. The braille slate which is

rectangular in shape may be used as an example with the border strip indicating the pathways.

7. Area of a rectangle

The product of the length and width of a rectangle is the area of the rectangle.

Therefore, area of a rectangle = length × width

$$\text{In symbols, A} = l \times b$$

$$= lb \text{ Sq. units}$$

Here, 'A' refers to the area, 'l' refers to the length, and 'b' refers to the breadth (or width) of the rectangle.

Note :

Here, A = lb and so lb = A

Then $b = \dfrac{A}{l}$

And also $l = \dfrac{A}{b}$

That is, length $= \dfrac{\text{Area}}{\text{Width}}$

and width $= \dfrac{\text{Area}}{\text{Length}}$

The idea may be explained to the child by demonstration using the rectangular sheet.

8. Area of a right angle triangle

A right angle triangle can be obtained by cutting across the diagonal of a rectangle. That is a right angle triangle is one of the two identical halves of a rectangle when it is cut across its diagonal. Hence the area of a right triangle will be equal to one half of the area of the rectangle.

Thus, area of a right angle triangle $= \dfrac{1}{2} \times$ base × height

In symbols, $A = \dfrac{1}{2} bh$ where 'b' is the base and 'h' is the height of the right angle triangle.

The idea may be explained to the child orally again through paper folding or through cutouts.

9. **Perimeter of a right angle triangle**

The perimeter of a right angle triangle is the sum of its three sides.

Eg. : *Find the perimeter of the right triangle whose sides are 3 cm, 4 cm and 5cm respectively.*

Perimeter of the right triangle = Sum of its three sides

= 3 + 4 + 5

= 12 cm

As the perimeter of the triangle is equal to the sum of its sides, the procedure followed in finding the perimeter of a square and the rectangle may be followed in the case of the triangle also.

10. **Area of four walls**

Area of four walls = Perimeter of the room $\times$ height

In symbols, A = ph, where 'p' is the perimeter of the room and 'h' is its height.

Eg.   : *The length, width and height of a room are 10 m, 6 m and 4m respectively. Find the area of its four walls.*

Given, length (l) = 10 m, width (b) = 6m and height (h) = 4m

Therefore, perimeter of the room,   p = 2 (l + b)

= 2 (10+ 6)

= 2 (16)

= 32 m

Area of the four walls   = p × h

                                          = 32 × 4

                                          = 128 m²

The idea may be explained to the child by providing a small box treating sides as length and width. Let him/her measure the sides using a thread and calculate the area of the walls using the above formulae.

Module 60

MEASURES OF VOLUME

1. Measures of volume

The space occupied by any object is called the volume of that object. Also, the measure of the inner space of a hollow solid is called its capacity.

The measure of length of a line segment is in terms of centimeter. The measure of area of a region or surface in terms of the unit is called the unit square. The volume of a solid is measured in terms of unit cube. To express the volume of solids, cubic centimeter or cm^3 is used.

Introduction of the concept of volume must be preceded by a clear explanation about the difference between area and volume. When the child is able to conceptualize the difference between the two, the idea of volume may be taught using three dimensional boxes and cylinders.

To enable the child to have a better understanding of the concept of volume, a bottle filled with water may be given for exploration. Then the water in the bottle may be poured to another container and now the child be told that the volume of the bottle is equal to the volume of water in the container.

MEASURES OF TIME

1. Measures of time

The smallest unit of time is normally the second, followed by the minute and the hour.

60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
7 days	=	1 week
28/29/30/31 days	=	1 month depending on the nature of month and the year.
12 months	=	1 year

The measures of time may be introduced orally with the provision of a tactile watch. When the child is comfortable with ideas of second, minute and hour, other related ideas such as number of hours in a day, number of days in a week, number of days and weeks in a month and the number of days, weeks and months in a year may be taught.

2. Ordinary year

The amount of time the earth takes to make one complete revolution about the sun is 1 year which is equivalent to 365 days. Thus an ordinary year constitutes 365 days.

The idea of an ordinary year may be taught to the child orally, explaining the number of days in each month.

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To enable the child to understand the idea of number of days in each month, the child may be asked to show his/her fist and say the months from January. Note that the thumb is not taken into consideration in calculating the number of days of a month. In the fist, January being the first month of the year stands for the bone that protrudes and the next month February in the slope. Now tell the child that all the months that protruded are with 31 days and all the months in the slope are with 30 days excepting February, which will be having either 28 days in the case of a ordinary year and 29 days in the case of a leap year. Let the child understand that both the months of July and August have 31 days.

3. **Leap year**

To be more accurate, the earth takes $365 \frac{1}{4}$ days to make one complete revolution about the sun. This extra $\frac{1}{4}$ day, making one day once in four years is added to February. Such a year is known as a leap year. Thus a leap year comprises 366 days. To know whether a particular year is a leap year or not, it is enough to divide the particular year by 4. If the year is exactly divisible by 4, then it is a leap year, otherwise it is not a leap year.

If the year taken into consideration is a century year divide it by 400, and in the process if the remainder happens to be zero then it is a leap year.

For example, 1600, 2000 are leap years while 1700, 1800, 1900 etc., are not leap years.

Note : 1. A decade is a period of 10 years.

2. A century is a period of 100 years.

3. A millennium is a period of 1000 years.

The idea of leap year may also be taught to the child orally, explaining the total number of days in the month of February in both the cases of an ordinary year and a leap year.

RADICALS

1. **Radical**

   The symbol ' ' indicates the square root of a number following the sign. In general, the nth root of 'r' is written as $\sqrt[n]{r}$ and the symbol 'n' is known as the radical sign where 'n' **is the order of the surd** and 'r' **is the radicand** and can also be written as $r^{1/n}$. The idea needs verbal explanation, in addition to the provision of relevant text material in Braille for the child to understand.

2. **Square number**

   A number is said to be a square number if it can be expressed as the product of two equal numbers. A square number is also called as a perfect square.

   Eg. : $4 = 2 \times 2$ Also, $4 = (-2)(-2)$

   $16 = 4 \times 4$ Also, $16 = (-4)(-4)$

   $144 = 12 \times 12$ Also, $144 = (-12)(-12)$

   Since a square number can be expressed as the product of two pairs of equal numbers, that is, one positive and the other negative, any square number will be having two square roots of the same magnitude but different in sign.

   The idea may be explained to the child orally.

3. **Square root**

A square number if expressed as the product of two equal numbers, then one of the equal numbers is said to be the square root of the given number.

Eg. : *16 = 4 × 4*

In the above example, since 16 can be expressed as the product of two equal numbers, namely 4, then 4 is the square root of 16.

The process of finding the square root of a number may be explained to the child with the help of an abacus.

Generally, if the square root of a particular number is asked than the positive root alone is to be mentioned.

The child must be clearly explained that any square number will be having two square roots, equal in magnitude but different in sign.

4. **Cube number**

If a number can be expressed as the product of three equal numbers, then the given number is said to be a cube number.

Eg. : 8 = 2 × 2 × 2

27 = 3 × 3 × 3

Since 8, 27, etc., can be expressed as the product of three equal numbers they are cube numbers.

The idea of a cube number may be explained through demonstration.

5. **Cube root**

When a cube number is expressed as the product of three equal numbers then one of the equal numbers is said to be the cube root of the given number.

Unlike square root, note that a negative number cannot be the cube root of a number.

Eg. : The product of 4 with 4 three times is equal to 64. That is, $4 \times 4 \times 4 = 64$.

Also note that $(-4) \times (-4) \times (-4) = -64$. Hence the cube root of 64 is 4 and the cube root of -64 is -4.

The idea of cube root may be explained to the child orally, providing additional input in the form of Braille text material.

ALGEBRAIC CONCEPTS

Algebra

The word 'Algebra' is derived from the Arabic word al-jabr. In Arabic, **'al' means 'the'** and **'jabr' means 'reunion of broken parts'.** Diophantus, a Greek mathematician is considered to be the father of Algebra. Algebra is a branch of mathematics in which **constants and variables** are used along with basic arithmetic operations. In other words, algebra may be considered as the **generalized arithmetic**.

1. **Mathematical statement**

    A statement is a meaningful combination of words. In addition to the words of the statement, if numbers are also used to make the statement then it becomes a mathematical statement.

    Eg. : *John is 10 years old now. What will be the age of him after 5 years?*

    Note that the above statement involves three numbers, of which 2 are known and the remaining one is unknown. Hence the statement is a combination of both words and numbers and hence it forms a mathematical statement.

    The idea of a mathematical statement may be explained to the child orally with as many examples as possible. After the provision of adequate examples the child may also be asked to give a few mathematical statements of his/her own.

2. **Place holder**

    Mathematical statements involve unknown numbers and different symbols are being used to denote these unknown numbers. Such symbols are known as place holders,

since they hold the places. 'x' is one such a place holder used to denote an unknown value.

3. Literals

In the expression of a mathematical statement, place holders are used to represent unknown numbers. **Letters like a, b, c, x, y, etc., are also used to represent unknown numbers in place of the place holders.** These letters, which are used to represent numbers are called as literals.

Eg. : *Consider the statement,* "*the product of 4 and another number becomes 20".*

Using literals, the statement can be rewritten as, 4 × b = 20. Here, 'b' is a literal used to denote an unknown value.

Note : The word literal is derived from "***litera***" *a Latin word meaning* "***a letter of the alphabet***"**.**

The idea may be explained to the child orally supported by relevant text material in Braille.

4. Constants

A quantity that takes a fixed numerical value is called a constant. The term 'constant' literally means a number or quantity that does not change its value. In Algebra too, the term means the same. Numerals are always constants, and in addition, certain other alphabets such as a, b, etc., can also be treated as constants depending upon the situations.

Eg. : *1,4,7, 15 etc., are all constants.*

The idea of constant can be taught to the child orally, explaining its literal meaning and its usage in mathematics.

5. Variable

A variable denotes a quantity (or letter or symbol) which is able to represent different numerical values. In other words, a variable denotes a quantity whose value changes depending upon the situations.

Generally, x, y, z are the three variables which are used most frequently. Depending upon the need, certain other alphabets are also treated as variables.

Note : 1. *All the numerals are constants.*

2. *To denote variables, the English alphabets from A to Z (or) a to z, excepting **o, i, j, k** are used.*

The concept can be taught to the child orally, and then focusing on the difference between a constant and a variable.

6. Power (or Exponent or Index) of a variable

The power of a literal indicates the number of times the base (variable) has been multiplied by itself.

Eg. : $a \times a \times a \times a = a^4$

In the above example the variable 'a' occurs four times and hence it can be denoted as a^4, which is to be read as 'a' to the power of '4'. Note that in a^4, a is called as the base and 4 is called the power. The term power is also called as **exponent or index.**

The idea may be explained to the child orally, focusing on the point that the amount of times a particular number is repeated becomes the power.

7. Coefficient

The numeral (or constant) which precedes the variable or the product of variables is called as the coefficient.

Eg. : *Consider 2x. Here the constant is 2 and the variable is x. Therefore the constant 2 is called as the coefficient.*

The idea may be explained to the child orally.

8. Algebraic Term

The **product of a variable and a constant** is said to be an algebraic term. In other words, a constant or variable or the combination of a constant and variables combined by means of multiplication (or division) is called a term or an algebraic term.

Eg. : 4, 35, -6 etc., are all constants

X, yz, -m^2, etc., are all variables

The **combination of the constants and the variables** like 4x, 3ab, -7yz are all **algebraic terms**.

After explaining the concepts of constant and the variable, the idea of an algebraic term be taught to the child through written demonstration. In the course of the teaching process, the child may be asked to give some examples for algebraic terms.

9. Like terms

Two or more terms which have the **same variable** or the **same product** of variables or **same division** of variables are called like terms.

Eg: 2x and - 4x ;   5 xy and 19 xy ; $\dfrac{4}{z}$ and $\dfrac{-15}{z}$

The above mentioned pairs form a set of like terms since the variables in all the cases are same. Note that the sign of the variable may differ in the case of like terms.

The idea may be explained orally and once the child acquires adequate familiarity, he/she may be asked to give some more examples.

10. Unlike terms

Two or more terms which have **different variables or different product of variables or different division of variables** are called as unlike terms.

Eg. : 3x and 5z ;   5ab and -6ac; 3m^2 and 3m^3

Note that in all the above mentioned cases the variables are different and hence they are all unlike terms.

As in the case of like terms, the idea of unlike terms may also be explained to the child emphasizing the difference between the two.

11. Addition of terms

Addition can be performed among algebraic terms only when they are like terms.

Eg. : *Add :   5x, 10 x, 7x*

Here all the three given terms are like terms and therefore, addition can be performed.

Therefore, 5x + 10x + 7x = (5 + 10 + 7) x

= 22 x

Note that the sum of two or more like terms is a term whose coefficient is the sum of the coefficients of all the like terms.

The concept may be explained to the child with examples with the provision of adequate exercises involving addition of like terms.

12. Subtraction of terms

As in the case of addition, subtraction among algebraic terms is also possible only when the terms are like terms.

Eg. : *Subtract : 14xy from 35xy*

In the above mentioned example the given terms are like terms, that is with the same product of variables x and y, and hence subtraction is possible.

Therefore, 35xy – 14xy = (35 – 14) xy

= 21xy

*Note that the difference between two like terms is a term with the same variables and whose coefficient is the **difference between the numerical coefficients** of the two like terms.*

The idea may be taught with written descriptions supported by necessary Braille text material. To enhance the understanding level of the child, he/she may be facilitated to perform a few sums on his/her own.

13. Equation

An equation is a **statement in which the values of two mathematical expressions are equal,** indicated by the sign $=$'.

Eg. : x + 5 = 3

2x + 7y − 3z = 21

The meaning and the significance of the symbol $=$' is to be emphasized initially and then sufficient number of examples are to be given.

14. Solution of an equation

Solution is a means of solving an equation. In other words, solution of an equation is the value of the unknown variable which when substituted, will satisfy the equation.

Eg. : Consider the equation, 2x + 4 = 10

On solving this equation,   2x = 10 − 4

$$2x = 6$$

Therefore, $x = \dfrac{6}{2}$

ie.,   x = 3

On substitution of the value of 'x' in the given equation,

2(3) + 4 = 10

6 + 4 = 10

10 = 10

That is, the given equation with an unknown variable is satisfied when the unknown variable is substituted with the value of 3.

Therefore, the solution of the given equation is 3.

The process of solving an equation is to be explained to the child with braille text material followed by the demonstration of a problem by the teacher and then asking the child to solve a few other problems.

15. **Inequality**

An inequality is a mathematical statement in which the two sides of the statement are not equal to each other. If the two sides of the statement are not equal to each other, then the possibilities are one of the sides must be either greater than or less than the other side. The symbol for greater than is $>$', and the symbol for less than is $<$'.

Eg. :　　　4 > 3

　　　- 435 < -2

　　　x + y < 10

　　　-x + 5y > 57

The idea of inequality may also be explained with Braille text, clearly narrating the difference between an equation and an inequality.

16. **Additive property of inequality**

The additive property of inequality states that if a number is added to both sides of an inequality, then the value of the inequality will not change.

That is, if a < b, then　a + c < b + c

　　　　if a > b, then　a + c > b + c

The property may be explained through Braille text.

17. **Multiplicative property of inequality**

The multiplicative property of inequality states that when an inequality is multiplied on both sides by a positive integer, then the value of the inequality is not changed.

That is, if 'c' is a positive integer, and

if a < b then ac < bc
if a > b then ac > bc

Note that if an inequality is multiplied on both sides by a negative integer then the inequality changes.

Also note that, if a < b, then -a > -b. Also, if c > d then, –c < -d (a, b, c, d are positive integers).

The idea may be taught with written examples supported by necessary text material in Braille. The idea that the negative sign of a number makes it smaller needs to be explained. The numberline is the best way to teach this idea. Keeping the zero as the mid point. Let the child understand that every additional unit on the positive side makes the number bigger (ie., 3 > 2) whereas every unit on the negative side makes it smaller (ie., –3 < –2) since more 'minus' means more 'dimunitive' value.

18. Metric system

The quantities used to find lengths, capacities, weights of things etc., are called measures. Several countries have their own system of measures. Of all the available systems the Metric system of measures is considered as simple and hence most of the countries in the world use the metric system of measures.

In Metric system, the basic unit of length is meter (m) the basic unit of weight is gram (g), and the basic unit of capacity is liter (l)

Metric system is also known as decimal system, as it has other sub units in powers of 10.

1000	100	10	1 (unit)	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Kilo	Hecto	Deca	Meter Gram Liter	Deci	Centi	Milli

The higher units are named by prefixing deca (means ten), hecto (means hundred) and kilo (means thousand). The lower units are named by prefixing deci (means a tenth), centi (means a hundredth) and milli (means a thousandth).

Conversion:

In the metric system, to convert a higher unit into a lower unit multiply it by powers of ten and to convert a lower unit into a higher unit divide it by powers of ten.

Note 1 : Linear measures (Measures of length)

Linear measures		Short form of writing
Millimeter	-	mm
Centimeter	-	cm
Decimeter	-	dm
Meter	-	m
Decameter	-	dam
Hectometer	-	hm
Kilometer	-	km

Note 2 : Conversion table

10 mm	-	1 cm
10 cm	-	1 dm
10 dm	-	1 m
10 m	-	1 dam
10 dam	-	1 hm
10 hm	-	1 km

Also,

100 cm	-	1m
1000 m	-	1 km

Measures of Weight

10 mg	-	1 cg
10 cg	-	1 dg
10 dg	-	1 g
10 g	-	1 dag
10 dag	-	1 hg
10 hg	-	1 kg

Also,

1000 mg	-	1 g
1000 g	-	1 kg
100 kg	-	1 quintal (q)
1000 kg	-	1 tonne (ton)

Measures of Capacity

10 ml	-	1 cl
10 cl	-	1 dl
10 dl	-	1 l
10 l	-	1 dal
10 dal	-	1 hl
10 hl	-	1 kl

All the ideas regarding metric system may be explained to the child through tactile tables supported by relevant text material in Braille, and once the child is clear with the ideas he/she may be asked to go to a shop and buy a few things, say for example, 500 grams of rice, 50 milliliters of shampoo and 2 meters of thread, without the assistance of others.

If the child is having the habit of going to the shop for purchasing some articles, then he/she must have the knowledge of the metric measures already. The task of the teacher is in associating the already familiar ideas to the current learning condition.

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19. Addition in metric measures

As the metric measures are based on powers of ten, addition and subtraction in metric system is just like addition and subtraction in number system.

Eg. : *Add : 6m 3 dm 5 cm ; 5m 4dm 5 cm and 5m 3 dm 2 cm*

m	dm	cm
6	3	5
5	4	5
5	3	2
17	**1**	**2**

That is, the result is   17m   1 dm   9 cm

The idea may be explained to the child that the changes in higher values are in terms of tens and therefore, the additions is just like regular additions.

The concept of borrowing numbers from the higher columns is similar to that of abacus and therefore use of abacus during this lesson will enhance the learning skills of the child.

20. Subtraction in metric measures

The process of subtraction in metric system is also just similar to addition.

Eg. : *Subtract 5 g 4 dg 3 cg   from 7 g 5 dg 6 cg*

g	dg	cg
7	5	6
5	4	3
2	1	3

Therefore the difference between the two quantities is 2 g 1 dg 3 cg

21. Multiplication in metric measures

As the metric measures are in powers of ten, multiplication in metric measures is the same as in the number system.

Eg. : *Multiply : 5 m 12 cm   by 4*

    5m 12 cm   =   5.12 m
Therefore, 5.12 × 4   =   20.48 m
                      =   20 m 48 cm

Note : Multiplication or division must be carried out only after converting the measures into a single unit, with or without a decimal point

22. Division in metric measures

As the metric units are in powers of ten, division in metric measures is also the same as in the case of number system.

Eg. : Divide 3 l 380 ml by 20

    3l 380 ml   = 3380 ml

    3380 ÷ 20 = 169

    Therefore, 3380 ml ÷ 20 = 169 ml

Division in metric measures may also be taught to the child through heaps of sticks of varying sizes. While the values are converted into highest measure in multiplication, the conversion to the lowest measure is to be made in division.

Every 10 small sticks may be replaced by one big stick and so on. This type of grouping numbers will help the child to understand formulation of frequency tables in statistics. This method facilitates discovery in the child.

23. Scientific notation

The scientific notation for a number is, that number written to the power of 10 multiplied by another number 'x' such that $1 \leq x < 10$.

In other words, the method in which very large and very small numbers are expressed using the powers of 10 is called scientific notation.

Eg. : *The diameter of the sun is 4567000000 feet = 4.567 $\times$ 10^9 feet.*

Wave length of red light is 0.00000216 cm = 2.16 $\times$ 10^{-6} cm.

The idea of scientific notation may be taught with small examples first with provision of adequate written material in Braille.

LAWS OF INDICES

1. Exponential form

The product of a number when repeated more than once can be expressed in the exponential form in which it is customary to write the number of times of repetitions as the power (or index or exponent). When the number is multiplied by the same number, the result is known as the square of that number and when the number is multiplied repeatedly thrice by itself, the result is the cube of the number.

For example, $5 \times 5 \times 5 \times 5 = 5^4$

Here 5 is the base and the number of times it has been repeated, that is 4 is said to be the exponent (or power or index).

The idea may be explained to the child through written Braille text.

2. Laws of indices

Law 1

The product of two or more numbers with the same base is equal to sum of the powers with the same base.

That is, in mathematical terms, if 'a' is the base and 'm' and 'n' are the powers, then, $a^m \times a^n = a^{m+n}$

Note that if the power is not mentioned then the power is 1.

Law 2

Let 'a' be the base and 'm' and 'n' be the powers.

If   m>n, then $\dfrac{a^m}{a^n} = a^{m-n}$

If   m<n, then $\dfrac{a^m}{a^n} = a^{n-m}$

Law 3

The **power of a power** is equal to the product of the powers.

$(a^m)^n = a^{mn}$

Law 4

The **power of a product** is equal to the product of the powers.

$(ab)^m = a^m \times b^m$

Law 5

The **power of a quotient** is equal to the quotient of the powers.

$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Law 6

Any number (other than zero) raised to the power zero is equal to 1.

$A^0 = 1 \ (A \neq 0)$

Law 7

$a^{-n} = \dfrac{1}{a^n}$

Note that a^{-n} is the multiplicative inverse of a^n.

These abstract ideas can be explained only through more concrete examples and written descriptions. Use of Braille text material describing these algebraic expressions is important. As indicated in module 1, effective learning of mathematics takes place when the teaching aids, descriptions, etc., are supplemented by Braille text materials.

ALGEBRAIC IDENTITIES

Algebraic identities

Algebraic identities are the generalized statements of a number of particular cases.

1. $(x+a)(x+b)$ $=$ $x^2 + (a+b)x + ab$

2. $(x-a)(x+b)$ $=$ $x^2 + (b-a)x - ab$

3. $(x+a)(x-b)$ $=$ $x^2 + (a-b)x - ab$

4. $(x-a)(x-b)$ $=$ $x^2 - (a+b)x + ab$

5. $(a+b)^2$ $=$ $a^2 + 2ab + b^2$

6. $(a-b)^2$ $=$ $a^2 - 2ab + b^2$

7. $(a+b)(a-b)$ $=$ $a^2 - b^2$

8. $(a+b+c)^2$ $=$ $a^2+b^2+c^2+2(ab+bc+ca)$

9. $(x+a)(x+b)(x+c)$ $=$ $x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

10. $(a+b)^3$ $=$ $a^3 + 3a^2b + 3ab^2 + b^3$

11. $(a-b)^3$ $=$ $a^3 - 3a^2b + 3ab^2 - b^3$

12. $a^3 + b^3$ $=$ $(a+b)(a^2 - ab + b^2)$

13. $a^3 - b^3$ $=$ $(a-b)(a^2 + ab + b^2)$

14. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ $=$ $a^3+b^3+c^3 - 3abc$

Upto second order, paper folding will be of use. For example, $(a+b+c)^2$ can be explained by creating a paper folding similar to

```
        a      b      c
     ┌─────┬──────┬──────┐
  a  │     │      │      │
     ├─────┼──────┼──────┤
  b  │     │      │      │
     │     │      │      │
     ├─────┼──────┼──────┤
     │     │      │      │
  c  │     │      │      │
     └─────┴──────┴──────┘
```

The higher order needs to be explained with written materials using the logic of second order. Sufficient practice must be given in enabling the child to recollect all the identities once they are taught. Some general principles may help to understand. Take for example $(a+b)^3$.

Here the products are of three types - purely involving 'a', purely involving 'b' and a product of 'a' and 'b' with different combinations. Let the child be helped to understand a pattern in the expansion.

$$(a+b)^2 = a^3 + 3a^2b + 3ab^2 + b^3$$

In the third order, the sum of the powers of the products should necessarily be 3. That is $3a^2b$ is a product of a & b, with the power of 'a' as 2 and power of b as '1'. Moreover, the power of the first variable decreases in the subsequent place values. That is when the power of one variable 'descends', the other value 'ascends'. That is why the third number is $3ab^2$, one power less than the previous one in the case of 'a' and one more in the case of 'b'. This logic can be applied in $(a+b)^2 = a^2 + 2ab + b^2$, etc.

POLYNOMIALS

1. Algebraic expression

Two or more algebraic terms connected by positive(+) or negative(-) signs is said to be an algebraic expression.

Eg. :   2x + 3y

4y - 5z

x + 4y - 9z

Having explained the relative terminologies such as **constant, variable** and **algebraic term** earlier, the idea of algebraic expression can be taught to the child without much difficulty.

2. Monomial

An algebraic expression having only one term is said to be **a monomial**.

Eg. :   2x, -4y, 5xy, -9xyz are all monomials as they contain only a single algebraic term. Note that eventhough the terms 5xy and -9xyz involve more than one variable they are monomials only.

The concept can be explained to the child with necessary illustrations in Braille.

3. Binomial

An algebraic expression with two terms is said to be a binomial.

Eg. :   x - y, 2y + 4z, 5xy - 7 yz are all binomials as they contain two algebraic terms.

The concept of adding a monomial to another monomial making it two monomials which is otherwise called 'binominal' needs to be explained so that the child can understand trinominal, polynominal, etc., easily.

4. **Trinomial**

An algebraic expression with three terms is said to be a trinomial.

Eg. : x + 2y - 3z,   - 2x + 7y - 9z, 2xy - 3yz - 8 zx   are all trinomials as they all contain three algebraic terms.

5. **Polynomial**

A function P(x) of the form, $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$ where $a_0, a_1, a_2, a_3, \ldots, a_n$ are real numbers and 'n' is a non-negative integer, is called a polynomial in 'x' with real coefficients. To state in simple terms, **an algebraic expression having four or more terms is called a polynomial**. It is customary to denote a polynomial as P(x).

Eg. : $P(x) = x^3 + x^2 - x - 5$

Here the algebraic expression contains four terms and hence it is called a polynomial.

In fact, **polynomial is a generic term denoting all the algebraic expressions**. A polynomial with a single term is called as monic polynomial or a monomial, and the idea is same for expressions containing higher number of terms also.

If more than one polynomial are to be dealt with at the same time, then they can be denoted as Q(x) and R(x) also.

With the description of monomial, binominal and trinomial, the child will be able to understand the concept of polynomial easily.

6. **Degree of a Polynomial**

The exponent in the term with the highest power is the degree of the polynomial.

In general, if $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0$ is a polynomial, $a_n \neq 0$, then 'n' is the degree of the polynomial.

In simple terms, the highest power of the variable occurring in the polynomial is said to be the degree of the polynomial.

For instance, if the highest power of the variable in a polynomial is 1 then the degree of the polynomial is 1, and the polynomial is also called as a linear polynomial.

Note :
1.   *If the highest power of the variable is 1, then the polynomial is **linear**.*
2.   *If the highest power of the variable is 2, then the polynomial is **quadratic**.*
3.   *If the highest power of the variable is 3, then the polynomial is **cubic**.*
4.   *If the highest power of the variable is 4, then the polynomial is **bi-quadratic**.*
5.   *If the highest power of the variable is 5, then the polynomial is **quintic**.*

The idea of order of a polynomial is to be explained to the child orally followed by necessary input in finding the order of a polynomial.

7.   Ascending order polynomial

If the terms of a polynomial are in the increasing order with respect to their degrees, then the polynomial is said to be in ascending order.

Eg. : *3 − 4x + 5x² − 7 x³*

As the child has already been exposed to the idea of ascending order involving numbers the concept of ascending order polynomial may be understood by the child without much difficulty.

8.   Descending order polynomial

If the terms of polynomial are in the decreasing order with respect to their degrees, then the polynomial is said to be in descending order.

Eg. : *6x⁴ − 5x³ + 4x² + 3x - 33*

As the child is already familiar with the idea of descending order involving numbers he/she may be able to understand the idea of descending order polynomial.

9. Negative of a Polynomial

For a polynomial P(x), its negative denoted as – P(x) is obtained by interchanging the signs of the terms of P(x).

For instance, if P(x) = x^2 + 5x + 9

then, –P(x) = – x^2 – 5x - 9

Note that, P(x) + [–P(x)] = P(x) – P(x) = 0

 The idea may be taught through exercises adopted for addition, subtraction, etc.

10. Value of a Polynomial

The value of a polynomial is obtained by replacing the variable in the polynomial by a numerical constant. That is, if P(x) is the given polynomial then the value of the polynomial when x = a is equal to P(a).

For instance, if P(x) = $2x^2$ + 5x - 20

Then the value of the polynomial when x = 2 will be,

P(2) = $2(2)^2$ + 5(2) – 20

$\qquad$ = 2(4) + 10 – 20

$\qquad$ = 8 + 10 – 20

$\qquad$ = 18 – 20

$\qquad$ = –2

Asking the child to perform a few sums on his own in addition to the oral explanation of the idea will enhance the understanding level of the child. As there are mostly visual ideas, use of braille text is very important.

11. Zero of a Polynomial

If the value of the polynomial is equal to zero, then the value of 'x' becomes the zero of the polynomial.

Eg. : *Let, P (x) = x² − 5x + 6*

When x = 1, the value is not zero, when x = 3, the value is not zero, but when x = 2, the value is zero. Therefore P(2) = 0.

If x = 2, then   P(2) = 2² − 5(2) + 6

$$= 4 - 10 + 6$$

$$= 10 - 10$$

$$= 0$$

Here, when x = 2, the value of the polynomial is equal to zero. Therefore, the zero of the polynomial P(x) is 2.

The idea may be explained to the child orally supported by necessary text material in Braille.

12. Zero polynomial

If all the coefficients of a polynomial including the constant term are zero, then the polynomial is said to be a zero polynomial.

Eg. : *P(x) = 0x² +0x - 0*

The idea may be explained orally supported by relevant text material in Braille.

13. Addition of polynomials

Sum of any two polynomials can be found by adding the corresponding like terms of the two polynomials.

Eg. : *Let, P(x) = 5x² − 4x +9*

Q(x) = 4x² + 7x − 7

Then, P(x) + Q(x) = (5x² − 4x +9) + (4x² + 7x − 7)

$$= (5x^2 + 4x^2) + (-4x + 7x) + (9 - 7)$$

$$= 9x^2 + 3x + 2$$

A thorough explanation of the idea followed by the demonstration of the process by the teacher may enable the child to understand the idea.

14. Subtraction of polynomials

Subtraction among two polynomials can be carried out by subtracting the corresponding terms of one polynomial from the other.

Eg. : Let $P(x) = 2x^2 + 5x + 10$

$Q(x) = x^2 + 3x + 7$

Then $P(x) - Q(x) = (2x^2 + 5x + 10) - (x^2 + 3x + 7)$

$$= (2x^2 - x^2) + (5x - 3x) + (10 - 7)$$

$$= x^2 + 2x + 3$$

The process followed in addition can be followed for subtraction also.

15. Multiplication of polynomials
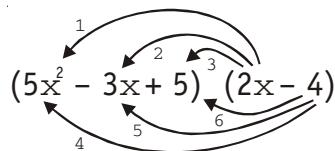
The product of two polynomials can be found by applying the distributive law and also the law of indices.

Eg. : *Find the product of $5x^2 - 3x + 5$ and $2x - 4$*

Therefore, $(5x^2 - 3x + 5)(2x - 4) = 10x^3 - 6x^2 + 10x - 20x^2 + 12x - 20$

$$= 10x^3 - 26x^2 + 22x - 20$$

A tactile diagram depicting the numbers may be helpful for the child to understand the sequence of the multiplication.

Another Method : Detached coefficient method
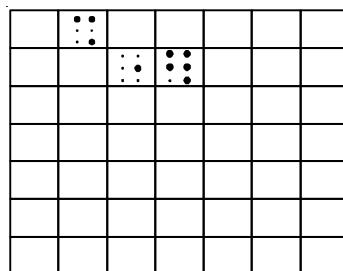
Eg. : Find the product of $5x^2 - 3x + 5$ and $2x - 4$

Solution :


```
5        -3        5
2        -4
_____
10      -6        10
        -20      12       -20
_____
10      -26      22       -20
_____
```

Therefore, the product is $10x^3 - 26x^2 + 22x - 20$

 Emphasis must be laid on the order in which the multiplication is to be performed. As this is a visual concept, the spatial presentation of this in Braille will help the child to understand the positions of the multiplied numbers. The cubarithm kind of mathematical board may also be helpful to develop this concept.

A panel or magnetic board with Braille inscribed numbers will be useful to understand the concept. In order to understand the correct rows and columns, separators in the form of thread may also be provided on the board.

16. Division of polynomials

The process of division of a polynomial f(x) by another polynomial g(x) is to find out two polynomials q(x) and r(x) such that f(x) = q(x)g(x) + r(x) where either r(x) = 0 or degree of r(x) is less than degree of g(x). Here f(x) is called the dividend, g(x) the divisor, q(x) the quotient and r(x) the remainder. In the process of division of f(x) by g(x) assume that the degree of the dividend f(x) is always greater or equal to that of the divisor g(x) because if degree of f(x) is less than the degree of g(x), then q(x) = 0 and r(x) = f(x). The applications of dividend, divisor, quotient

and remainder concepts used in abacus may be used here to explain the division of polynominals.

Eg. : *Divide $x^2 + 5x + 6$ by $x + 2$*

$$x + 2) \ x^2 + 5x + 6 \ (x + 3$$
$$\underline{x^2 + 2x}$$
$$3x + 6$$
$$\underline{3x + 6}$$
$$\underline{0}$$

Therefore, quotient = x+3 and remainder = 0

Another Method : Detached coefficient method

Eg. : Divide $x^2 + 5x + 6$ by $x + 2$

$$
\begin{array}{cc|ccc}
 & & 1 & 3 & \\
\hline
1 & 2 & 1 & 5 & 6 \\
 & & 1 & 2 & \\
 & & \hline & & \\
 & & 3 & 6 \\
 & & 3 & 6 \\
 & & \hline & \\
 & & 0 & \\
\end{array}
$$

Therefore, Quotient = x + 3   Remainder = 0

The idea may be explained to the child by presenting the procedure spatially on a Braille sheet supported by necessary explanation. The mathematical board may be useful for this too.

FACTORIZATION

1. Factorization

The process of expressing a polynomial as the product of two or more simpler polynomials is called as factorization.

Eg. : *x^2 − 4 = (x+2)(x-2)*

Therefore in this case x+2 and x-2 are the factors of x^2 − 4 as the product of x+2 and x-2 is equal to x^2 − 4.

*Note that x^2 − 4 is a second degree polynomial, while x+2 and x-2 are first degree polynomials. The process of factorization is also known as the **resolution into factors**.*

The idea may be explained using the examples of multiplication and division concepts used in abacus, followed by a demonstration of the process in factorizing.

2. Square root

As the concept of square root involving numbers is already known to the child, he/she may be taught the idea of finding the square of polynomials. As the detached coefficient method of finding square root is considered to be simpler than the usual method, the child may be taught the detached co-efficient method first using the mathematical board / cubarithm. Once the child is through with the detached coefficient method the other method may also be taught.

Find the square root of $9x^4$ − $42x^3$ +$19x^2$ + 70x +25

Solution:

```
                   3    -7   -5
              ┌─────────────────────────
         3    │ 9   -42  19   70   25
              │
              │ 9
              │ ─────────────
      6   -7  │      -42  19
              │
              │      -42  49
              │      ──────────
   6  -14  -5 │           -30  70   25
              │
              │           -30  70   25
              │           ─────────────
              │                 0
              │           ─────────────
```

Therefore, $\sqrt{9x^4 - 42x^3 + 19x^2 + 70x + 25} = 3x^2 - 7x - 5$

In the above example, the coefficients viz., 9, -42, 19, 70 and 25 are detached from the polynominal and written in order horizontally. Consider the first coefficient 9. Since 9 is a perfect square write its square root 3 in the place meant for quotient and also to the left of 9. Multiply 3 with 3 and write the product 9 below 9 and subtract. The result is zero. Now take the next two coeffecients -42 and 19. Double the quotient 3 and write the value 6 to the left of -42. Identify a suitable number which when multiplied by 6, gives -42.   In this case the number is -7. Write -7 in the quotient and to the left of -42 also. Now multiply and subtract -7 x 6 = -42 and -7 x -7 = 49. Subtract -42 and 49 from -42 and 19 respectively. Repeat the procedure till the remainder is zero. Note that if the remainder is zero then the polynominal is a perfect square. Otherwise, the polynominal is imperfect.

3. Linear equation in one variable

An algebraic equation with a single variable whose power is equal to 1 is termed as a linear equation in one variable.

Eg. : *4x – 3 = 2*

Here, the equation is with a single variable whose degree is 1 and hence it is a linear equation in one variable.

The idea may be explained to the child through the mathematical board.

4. Linear equation in two variables

An algebraic equation with two variables and the highest power of the variable as 1 is termed as a linear equation in two variables.

Eg. : 2x + 3y = 5

Here the equation is with two variables x and y, and also the highest power of the variables is 1 and hence it is a linear equation with two variables.

The idea may be explained to the child orally supported by relevant text material in Braille.

5. Solution of linear equation in one variable

Any **numerical value which satisfies the equation** is termed as the solution of the linear equation in one variable.

Eg. :

Consider the equation, 2x – 1 = 3

2x = 3 +1

2x = 4

x = 2

The idea may be explained to the child through exercises.

6.   Solution of simultaneous linear equations

The numerical values of the two unknown variables in a pair of linear equations in two variables are said to be the solution of the equations.

Eg. :

Consider,   x + y = 5   -   (1)

x – y = 1   -   (2)

Adding equations (1) and (2),

$$2x = 6$$

$$x = 3$$

Substituting the value of x in (1)

$$3 + y = 5$$

$$y = 5 - 3$$

$$y = 2$$

Therefore the solution of the two simultaneous equations is (3, 2)

The idea may be explained to the child through spatial forms of exercises. The process of solving simultaneous equations is to be emphasized more than once until the child is clear with the idea.

7.   Sequence

Numbers which follow a definite pattern are said to form a sequence.

Eg. : 1, 2, 3, 4, 5, ... the difference between numbers is '1'

2, 4, 6, 8, 10, ... the difference between numbers is '2'

1, 4, 9, 16, 25, ... the difference between squares of 1, 2, 3, 4, 5, etc.

Note : Each element in a sequence is called a term. The first term is denoted as t_1, the second term as t_2 and so on.

As this is a non-visual logic, the child with visual impairment can understand through adequate description.

8. **Finite sequence**

   If the last term of a sequence is known then it is called a finite sequence. The idea may be taught orally supported by relevant text material in Braille.

   Eg. : 3, 6, 9, ..., 45

9. **Infinite sequence**

   If the last term of the sequence is not known then the sequence becomes an infinite sequence.

   Eg. : 5, 10, 15...,

10. **Series**

    The **sum of the terms of** a sequence is called a series.

    Eg. : 5 + 10 + 15 + ... + 50

11. **Finite series**

    If the last term of the series is known it is a finite series. In other words a finite series terminates at a particular term.

    Eg. : 2 + 4 + 6 + ... + 100

12. **Infinite series**

    If the last term of a series is not known then it is an infinite series. In other words, an infinite series goes on and on.

    Eg. : 4 + 8 + 12 + ...

13. **Arithmetic Progression**

    A sequence in which the **difference between any two consecutive terms is equal** is called an Arithmetic sequence or an Arithmetic progression, and is shortly denoted as A.P.

    Consider the Arithmetic Progression

1, 4, 7, 10, ...

The difference between the second and the first term is 4 – 1 = 3. The difference between the third term and the second term is 7 – 4 = 3.

Also $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 3$.

Since the differences are the same, the difference is called the common difference and is denoted as 'd'.

14. Common difference
In an Arithmetic progression the difference between any two terms which is same all through the sequence is called as the common difference and is normally denoted as 'd'.

15. General term
The terms of an A.P are generally denoted as t_1, t_2, t_3, t_4,... That is, the first term is t_1, the second term is t_2 etc., and the last term is denoted as t_n. The **last term of the A.P is called as the general term** and can be found using the formula $t_n = a +(n-1)d$.

Note that if the last term of a sequence is known, then the number of terms in the sequence denoted as 'n' can be found using the formula, $n = \left(\dfrac{l-a}{d}\right) + 1$, where 'n' is the number of terms in the sequence, 'l' is the last term, 'a' is the first term and 'd' is the common difference.

As this is a simple arithmetic, the child can be taught by presentation of this in Braille format.

16. Increasing sequence
The common difference of an arithmetic progression is normally denoted as 'd'. If the value of 'd' is positive, then the sequence is an increasing sequence.

Eg. : 1, 4, 7, 10, ...

17. **Decreasing sequence**

   If the value of 'd' the common difference is negative, then the sequence is a decreasing sequence.

   Eg. : 10, 5, 0, –5, –10, ...

18. **Sum of an AP**

   The sum of the 'n' terms of an A.P can be found using the formula,

   $$S_n = \frac{n}{2}[2a + (n-1)d]$$

   where, S_n denotes the sum

   n is the total number of terms

   a is the first term

   d is the common difference

   If the last term of the sequence, denoted as 'l' is known, then the sum of the A.P is,

   $$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + \{a + (n-1)d\}] = \frac{n}{2}[a + l]$$

   where 'l' is the last term of the sequence.

   Eg. : Consider the A.P.   2, 4, 6, 8, ... 24

   Here the first term a=2

   Common difference d = 4-2=2

   Last term, l = 24

   ∴ Number of terms,

   $$= \frac{24 - 2}{2} + 1$$

   $$= \frac{22}{2} + 1$$

   $$= 11 + 1$$

$$n = 12$$

$$\text{Sum of A.P., } 2 + 4 + 6 + \ldots + 24 = \frac{12}{2}(2+24)$$

$$= 6\,(26)$$

$$2 + 4 + 6 + \ldots + 24 = 156$$

The idea needs to be explained through linear presentation of the text material, laying emphasis on the knowledge of the different symbols used in the formula.

19. Geometric Progression (GP)

Sequences in which the **ratio of each term to its predecessor** is always same are called as geometric sequences or geometric progressions, denoted as GP.

Eg. :   1, 2, 4, 8, …

           5, 15, 45, 135, …

The idea needs verbal explanation supported by relevant text material.

20. Common ratio

In a GP the ratio of each term to its predecessor which is always same is called as the common ratio and is normally denoted by the letter 'r'.

21. General term of a GP

The terms of a GP are normally denoted as t_1, t_2, t_3, t_4, … and the nth term is denoted as t_n. The nth term of a G.P is called as the general term of a GP and can be found using the formula,

$$t_n = ar^{n-1}$$

The idea may be explained through written material in Braille in linear format.

Eg. : Assume that the sequence is 1, 3, 9, 27, …. and has 6 terms. The general term which is the last of the series can be found out using the formulae.

$$t_n = 1 \times 3^{6-1}$$
$$= 1 \times 3^5$$

$$= \quad 3 \times 3 \times 3 \times 3 \times 3$$
$$= \quad 243$$

22. Finite GP

A GP whose last term is known is said to be a finite GP.

Eg. : 1, 2, 4, 8, 16, ... 256

Here the last term is known and hence it is a finite GP.

23. Infinite GP

A GP whose last term is not known is said to be an infinite GP.

Eg. : 1, 3, 9, 27, 81, ...

Here the last term is not known and hence it is an infinite GP.

24. Sum of the terms of a GP

The sum of the first 'n' terms of a GP can be found using the formulae:

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad \text{if } r > 1$$

$$= \frac{a(1 - r^n)}{1 - r} \qquad \text{if } r < 1$$

$$= \quad na \qquad \text{if } r = 1$$

Usually, the formulae may be memorized. In order to understand the structure of the formulae, they can be presented in both linear and spatial forms.

25. Sum of the first 'n' natural numbers

The sum of the first 'n' natural numbers can be found by using the formula,

$$1+2+3+4+5+6+ \; ... \; + n = \frac{n(n + 1)}{2}$$

where 'n' is the number of terms in the sequence.

26. Sum of the squares of the first 'n' natural numbers

The sum of the squares of the first 'n' natural numbers is found by the formula,

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots + n^2 \ = \ \frac{n\,(n+1)\,(2n+1)}{6}$$

where 'n' is the number of terms in the sequence.

27. Sum of the cubes of the first 'n' natural numbers

The sum of the first 'n' natural numbers can be found by the formula,

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \ldots + n^3 \ = \ \left\{ \frac{n\,(n+1)}{2} \right\}^2$$ where 'n' is the number of terms in the sequence.

Memorisation of mathematical formula is inevitable. However, the child be assisted to understand it first. Spatial presentation of this on braille paper is necessary to enable the child to explore and understand.

28. Synthetic division

Synthetic division is the simplified form of ordinary division. In synthetic division the coefficients are taken separately and then divided by the constant in the binomial.

Eg: Divide $x^2 + 5x + 6$ by $x - 2$

$$
\begin{array}{c|ccc}
2 & 1 & 5 & 6 \\
  &   &   &   \\
  & 0 & 2 & 14 \\
  & \hline
  & 1 & 7 & 20 \\
\end{array}
$$

Quotient = x + 7   Remainder = 20

Detach the coefficients 1, 5 and 6 from the quadratic expression x2 + 5x + 6 and write them in order horizontally. Since the diviser is $x - 2$ the coefficients must be divided by 2. *(Please note that the sign of the constant in the divisor is changed)*. Note that unlike normal division where the products are subtracted, in synthetic division the products are to be added. Also as a common rule, in the first step, zero is to be added with the first coefficient. Thus write 2 to the left of the coefficients and then add zero with the first coefficient '1'. The sum is 1. Now multiply 1 with the divisor 2.

Write the product 2 below 5 and add. The sum is 7. Multiply 7 with 2 and write the product 14 below 6. The sum is 20. Hence in this case the remainder is 20 and the quotient is $x + 7$.

29. Remainder theorem

When a polynomial P(x) of degree greater than 1, is divided by a binomial (x-a) where 'a' is a real number, then the remainder is P(a).

Eg. :

Let $P(x) = x^2-5x+6$

Dividing P(x) by x-1

Then $P(1) = 1^2 - 5(1)+6$

$= 1-5+6$

$= 7-5$

$P(1) = 2$

i.e., the remainder when $x^2 -5x+6$ is divided by x-1 is 2.

Unlike in the synthetic division where both the quotient and the remainder can be found, in order to find the remainder alone the remainder theorem may be used. In the above example the polynominal $P(x) = x^2 -5x -16$, which has to be divided by x −1. Consider x −1 = 0 $\Rightarrow$ x = 1. Hence in P(x), x has to be replaced by 1, thus finding P(1). Replacing x by 1 and simplifying leads to the value P(1) = 2. Thus the remainder when $x^2 − 5x + 6$ is divided by x−1 is 2.

30. Factor theorem

When a polynominal P(x) of degree greater than 1 is divided by a binominal (x-a), and if P(a)=0 then (x-a) is a factor of a polynominal P(x).

Eg. :

Let $P(x) = x^2 +5x+6$

Dividing P(x) by x+2

Then P(-2)= $(-2)^2$ +5(-2)+6

= 4-10+6

= 10-10

P(-2) = 0

∴ x+2 is a factor of x^2+5x+6

31. Solution of a quadratic equation by formula

Any quadratic equation of the form $ax^2+bx+c=0$ can be solved by using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving of any quadratic equation will lead to two values which satisfy the given equation, and these two values are called the roots of the equation.

First of all, the formula needs to be memorized. The idea may be taught orally supported by relevant text material in Braille. Adequate practice must be given in enabling the child to familiarize with the formula and knowing its components.

32. Nature of roots

The two roots of an equation can be real and equal, real and unequal or imaginary. It is not necessary to solve an equation to find the nature of its roots. The nature of the roots can be judged by finding value of $\Delta = b^2 - 4ac$, which is called the discriminant.

33. Discriminant

Discriminant is a value which reveals about the nature of the roots of a quadratic equation. Discriminant is denoted by the symbol Δ and its value is $b^2 - 4ac$. Note that if,

b² – 4ac >0 then the roots are real and distinct.

b² – 4ac =0 then the roots are real and equal.

b² – 4ac <0 then the roots are unreal.

Note : 1. If b² – 4ac is a perfect square, the roots are rational and distinct.

2. If b² – 4ac is not a perfect square, the roots are irrational and distinct.

The idea may be taught through practice in writing.

34. Relation between the roots

Let $ax^2 + bx + c = 0$ be any given quadratic equation. Solving of the equation will lead to the two roots which are,

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Note that $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Also, if the roots of an equation are known then the equation can be found using the formula, x^2 - (sum of the roots) x + (product of the roots) = 0

The idea may be taught by providing braille text.

Module 68

LOGARITHM

Logarithm

The concept of logarithm was introduced by a Scottish mathematician John Napier (1550 - 1617). John Napier together with Henry Briggs (1561-1631) constructed the ready to use logarithmic tables which are also known as ' common logarithms'. Note that the base of all the common logarithms is 10. **Logarithm is another form of expressing a number in exponential form.**

1. **Logarithm**

   The logarithm of a number to a given base is the power to which the base is to be raised to equal the given number.

   Consider the expression in exponential form, $a = x^m$. In logarithmic form the given expression can be rewritten as, $\log_x a = m$.

   Note : 1. *The base must always be positive and should not be equal to 1.*

   2. ***Logarithms are defined only for positive real numbers and not defined for 0 and negative numbers.***

   Eg.: Find the value of $\log_5 125$

   Let $\log_5 125 = x$

   Therefore in exponential form, $5^x = 125$

   That is, $5^x = 5^3$

Therefore, x = 3

$\log_5 125 = 3$

The idea of logarithm is to be explained with examples prepared in braille format.

2. Laws of logarithm

Product rule

The logarithm of a product of positive numbers is equal to the sum of the logarithm of the factors.

In symbols, $\log_x(AB) = \log_x A + \log_x B$

Quotient rule

The logarithm of the quotient of two positive numbers is equal to the difference between logarithm of the numerator and logarithm of the denominator.

In symbols, $\log_x (A/B) = \log_x A - \log_x B$

Power rule

The logarithm of a number raised to a power is equal to the power times logarithm of the number.

In symbols, $\log_x (A^p) = p.\, \log_x A$

Change of base rule

$\log_x A = \log_y A \times \log_x Y$

The above rules may be explained orally with the assistance of text material in Braille.

GEOMETRY - CIRCLE

1. **Circle**

A circle is a geometrical figure in which the distance from the centre to the circumference is equal at all points. In other words, a circle is the path traced by some continuous points which are at a constant distance from a fixed point, the centre.
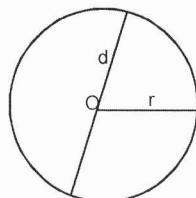
A ring / bangle may be given to the child and he/she may be asked to explore its shape. Simultaneously, the concepts of centre, radius, diameter, chord, tangent, secant can also be taught by using a thin stick along with the ring.

Note : 1. The diameter is the longest chord of the circle.

      2. Diameter = 2 (radius)

To enable child to understand the idea better, provide a small string in which a nail is attached to one of its ends. Fix the nail in the ground. On the other end of the string a small stick is to be tied. Now rotate the string all around the fixed nail keeping the string straight. Observe that a circle is obtained, in which the marking of the stick is the circumference, the point at which the nail is fixed is the center and the length of the string is the radius. When the child performs this activity by himself, all the ideas related to circle may be understood clearly.

The ideas may also be explained with the help of an embossed circle prepared with a sheet of paper.
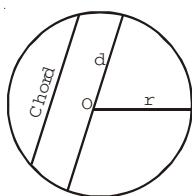
2. **Parts of a circle**

   Centre, radius, diameter, chord, segment, arc, sector and circumference are the different parts of a circle.

   Provision of a circular ring may enable the child to conceptualize the outer circumference of a circle.

   Further, when the child is constructing the circle by himself using the nail, thread and stick all the parts of the circle will certainly be clear as the activity is performed by the child himself. Paper folding exercise may also be adopted to teach these concepts. (Refer the section on Creative Mathematics).
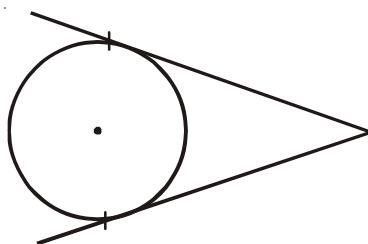
3. **Tangent**

   A line which touches the outer circumference of a circle at one single point is called the tangent. The idea of a tangent may be explained to the child with the assistance of a circular ring along with a small wooden strip, by making it touch the outer circumference of the ring at a single point.

4. **From a point outside of a circle two tangents can be drawn**

   Place the circular ring on a table. Take two wooden strips of the same length and arrange the strips in such a way that they form two tangents to the ring originating from the same point. On exploration, the child will be able to understand that from a point outside a circle two tangents can be drawn.
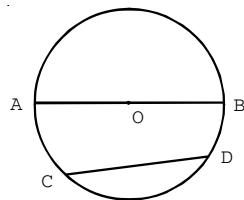
5. **Tangents drawn from a point outside a circle are of equal length**

   The lengths of the two tangents drawn from a point outside a circle will always be of equal length.

The procedure followed in the above case may be followed as it is, and when the child is allowed to measure the lengths of the strips, he/she will understand that the tangents drawn from a point outside a circle are of equal length.

6. Chord
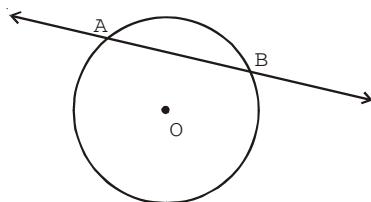
A chord of a circle is a line segment whose end points lie on the circle. The diameter is also a chord. If the chord passes through the center of the circle then it is called as the diameter. Note that the diameter is the longest chord of the circle.

The idea of chord may be taught with the aid of a circular ring and a suitable wooden strip. Embossed diagrams may also be used to enable the child to understand the idea better. Paper folding is also useful.

7. Secant

A line which intersects the circle in two points is called the secant of the circle. The procedure followed in teaching the idea of chord may be followed to teach the idea of secant also.

8. The value of 'π'

The symbol π is used to find out the area and the circumference of a circle. Approximately the value of π is equivalent to $\frac{22}{7}$ or 3.1416...

The idea is to be explained orally, supported by the explanation regarding the usage of the value of π in finding the circumference and area of a circle.

9. Circumference of a circle
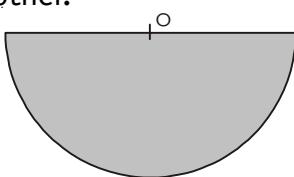
The length of the outer boundary of a circle is called its circumference. The circumference of a circle can be found using the formula 2 π r, where 'r' is the radius of the circle.

Provide the child with a circular ring along with a piece of thread. Explain the idea orally and then allow the child to measure the length of the outer boundary of the circle with the help of the thread and measure the length of the rope using a scale to find out the circumference of the circle.

10. Semi circle

Semicircle means half of a circle i.e., both the parts are exactly identical. A thick sheet of paper cut in the form of a circle may be given to the child to explore its shape. Later, the same circle may also be cut across its diameter to make two semicircles which are identical to each other.

To enhance the understanding level of the child, a sheet of paper in the shape of a circle may be given to the child with an embossed marking of its center. Ask the child to fold the paper along its center to form a diameter. Once folded, the circle becomes a semicircle with the edge of the circular paper as the diameter of the semicircle.

11. Area of a circle

Area of a circle can be found using the formula, A = π r^2 square units. The child needs to be taught the formula clearly enabling him/her to know about the value of π which is fixed and the value of 'r' which varies from circle to circle. Adequate practice may be given to enable the child to find out the areas of circles with different radii.

12. Area of a semicircle

A semi circle being exactly one half of a circle, the area of a semi circle is also exactly one half of the area of a circle of the same radius.

Hence, area of a circle = π r^2 square units

Therefore, area of a semi circle = $\dfrac{\pi r^2}{2}$ square units

The idea may be explained to the child orally along with the provision of a sheet of paper cut in the form of a semicircle.

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13. Perimeter of a semi circle

Perimeter of a semi circle = Half of the circumference of the circle + Twice the radius

= π r + 2r units

Note   :   The ratio of the circumference to the diameter of any circle is a constant.
The constant is denoted by π.

Once the child is provided a paper cutting of a semi circle then he/she will be able to understand that the circumference of the semicircle is equal to half the circumference of the circle plus twice the radius of the circle (which is nothing but the diameter).

14. The ratio between the circumference and diameter of a circle is constant
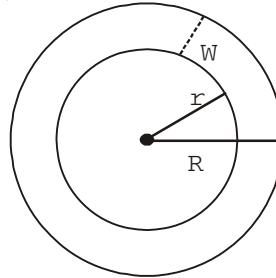
$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

Therefore, Circumference = π × diameter

The idea may be explained orally with the assistance of necessary text material in Braille.

15. Concentric circles

Two or more circles having the same centre but different radii are said to be concentric circles.

Note : Consider two concentric circles with the same centre 'O' and different radii. Let 'R' be the radius of the outer circle and 'r' be the radius of the inner circle. Let the distance between the circumference of the two circles, the width be 'w'. Then the relation between the three units R, r and w is given by,

R = w + r

r = R − w

w = R - r
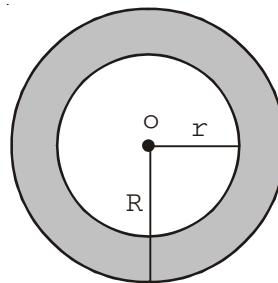
Two or more embossed concentric circles be prepared on a sheet of paper and the child be asked to explore. Tactually, the child can observe that all the circles have the same centre but different radii thus enabling him/her to understand the concept of concentric circles.

16. Circular ring

If two circles with the same center and different radii are drawn on the same plane, then the portion between the two circles is called as 'circular ring'.

The procedure adopted for explaining the idea of concentric circles may be repeated with the additional information on the idea of circular ring, which is nothing but the area formed between the two concentric circles.

17. Area of a circular ring

Area of a circular ring = $\pi (R^2 - r^2)$, where R is the radius of the outer circle and r is the radius of the inner circle.

As the child is already familiar with the idea of circular ring, the formula for finding the area of the circular ring may be explained orally. To enhance the understanding level of the child, he/she may be asked to find out the area of a few circular rings on his/her own after the demonstration by the teacher.
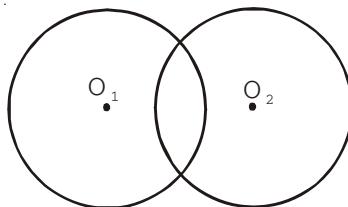
18. Intersecting circles

Two circles having some part of their area common between them are called as intersecting circles. In other words, two circles intersecting each other at two points are said to be intersecting circles.

Let r_1 and r_2 be the radii of two circles and let 'd' be the distance between their centers.

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That is, $r_1 + r_2 > d$

Therefore for intersecting circles, sum of the lengths of radii is greater than the distance between their centers.
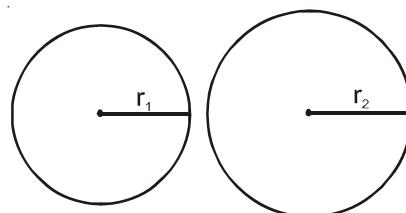
Two embossed intersecting circles may be prepared on a sheet of paper and the child may be facilitated to explore. Once the child conceptualizes the idea of intersecting circles the distance between their radii may be explained orally. The child can also measure the radius of each circle separately, add them, and measure the distance between the centres and realise that $r_1 + r_2 > d$.

19. Non-intersecting circles

Two circles which neither touch nor intersect each other are called non-intersecting circles.

If r_1 and r_2 denote the radii of the two non-intersecting circles and 'd' is the distance between their centers then, $r_1 + r_2 < d$. Therefore, for two non-intersecting circles, the sum of the lengths of their radii is less than the distance between their centers.

Two embossed non-intersecting circles be prepared on a thick sheet of paper and child be provided with that. On exploration the child will be able to understand that the sum of their radii is less than the distance between their centers.
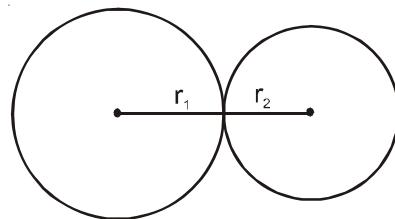
20. Circles touching externally

Two circles touching each other externally at a single point are said to be circles touching externally.

If r_1 and r_2 denote the radii of the two circles touching externally and 'd' is the distance between their centers then, $r_1 + r_2 = d$.

The child may be provided with two embossed circles touching externally and he/she may be facilitated to explore the distance between their centers. On exploration assisted by the explanation by the teacher, the child will be able to understand the concept.
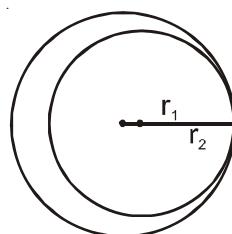
21. Circles touching internally

Two circles touching each other internally at a single point are said to be circles touching internally.

If r_1 and r_2 denote the radii of the two circles touching internally and 'd' is the distance between their centers then, $r_1 - r_2 = d$.

Two embossed circles touching internally may be provided and the child be allowed to explore, and now the child will be able to realize that the distance between their centers is equal to the difference of their radii.

Note : 1. If $r_1 - r_2 < d$, the circle with smaller radius will lie inside the larger circle.
2. If $r_1 - r_2 > d$, the two circles lie seperately.
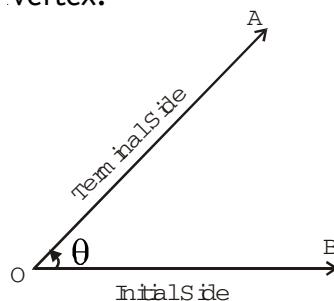
Module 70

ANGLES

1. Angle

An angle is a measure formed by two rays with a common starting point. In other words, the space between two lines or surfaces that meet at a point is said to be the angle. The two lines are also called as the initial side and the terminal side and the common point is called the vertex.
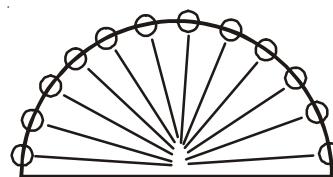
A sheet of paper in which two creases which originate from the same point may be created and given to the child. On exploration, the child might be able to understand that an angle is formed between the two lines.

2. Measurement of an angle
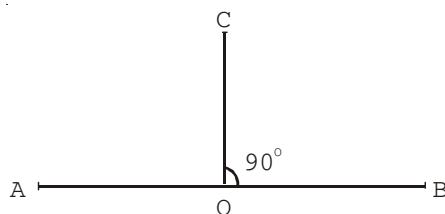
The rotation made by the ray from its initial side to the terminal side can be measured in terms of degrees which will be the measure of the angle formed.

The idea needs to be explained with illustrations. A protractor in which adaptations (with a hole or glue) may be made so that every fifth degree can be felt by the visually impaired child to measure the angle.

3.   Right angle

Any angle which measures exactly 90^0 is said to be a right angle. The idea can be taught by using the paper folding. Take a sheet of paper and fold it to make two halves. Now fold it once again to halve the paper on the other way. The two creases formed will make an angle of 90° between them.

The edges of a Braille slate or Braille paper normally make an angle of 90°. The child needs to be given hands on experience through the objects which he/she is familiar with.
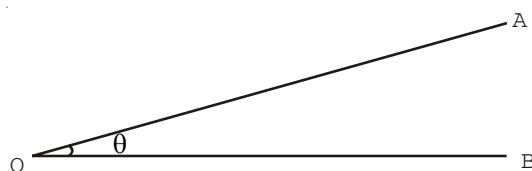
4.   Zero angle

If the initial side and terminal side coincide, then the angle formed is said to be a zero angle.

The idea of zero angle is totally abstract in nature. Place a sheet over the other and ask the child whether or not any angle is formed. Such a condition be explained as Zero angle.
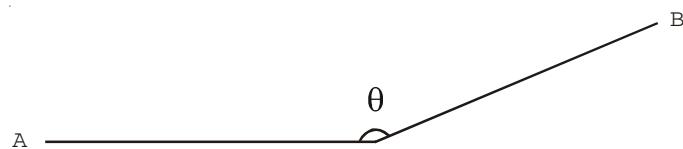
5.   Acute angle

Any angle which measures greater than zero degree and less than 90° is called an acute angle. Explain the concept clearly and then enable the child to make an acute angle on a sheet of paper by folding it accordingly. Since the child himself is performing the activity he/she will be able to understand the idea clearly.

6.   Obtuse angle

Any angle whose measure is greater than 90^0 and less than 180^0 is called an obtuse angle.

The procedure followed in teaching the idea of acute angle may be followed to enable the child to understand the idea of obtuse angle also.
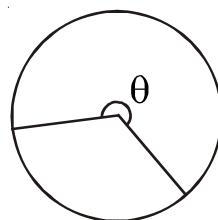
7. Straight angle

An angle which measures exactly 180^0 is called a straight angle. The idea needs to be explained orally, in addition to the provision of a sheet of paper in which the sides always form a straight angle.
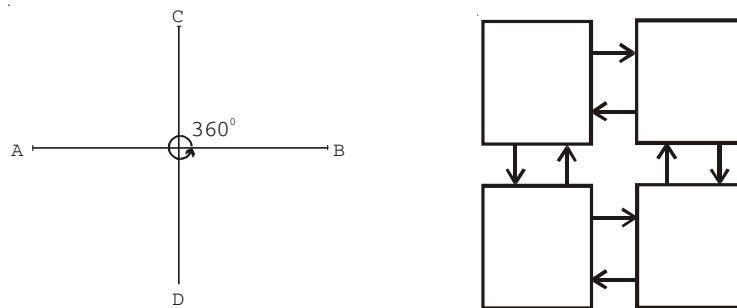
8. Reflex angle

Angles which measure greater than 180^0 and less than 360^0 are called as reflex angles. The idea may explained to the child through a tactile diagram.

9. Angle at a point

An angle which measures exactly 360^0 is called the angle at a point.
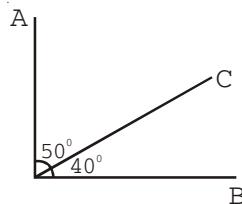
To enhance the understanding level of the child, provide him/her with four equal pieces of paper which are torn from a sheet, by folding it both vertically and horizontally. Facilitate the child to understand that any corner of the four pieces form a right

angle. Enable the child to affix the four pieces together, so that no space is left in between them. Also teach the child that when the four right angles are put together, with no space left in between then the angle so formed will naturally be equal to 360⁰ which is the angle at a point. The child may also be made aware that two straight angles will also constitute an angle at a point.

10. Complementary angles

Two angles are said to be complementary to each other if their sum is equal to a right angle and one angle is said to be the complement of the other.
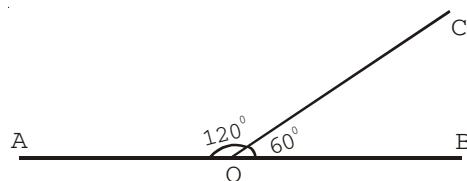
Eg. : $40° + 50° = 90°$

In the above example, the sum of $40°$ and $50°$ is equal to $90°$ and hence, $50°$ is the complement of $40°$ and vice-versa.

A sheet of paper which form an angle of $90°$ at its vertices be torn into two, at one of the vertices, each measuring some unknown angle. The two angles thus formed can be measured and the child be explained that the two angles are complement to each other, since their sum is equal to a right angle.

11. Supplementary angles

If the sum of any two angles is equal to a straight angle or $180°$ then the two angles are said to be supplement to each other.

A sheet of paper, neatly cut, which makes an angle of $180°$ at its sides is torn at one of its sides, so that one straight angle is divided into two. Now, on measuring the two

angles separately, the child understands fact that the two angles add up to a straight angle thus making the two angles supplement to each other.

12. **Linear pair**

   If the sum of two adjacent angles is equal to 180° or a straight angle then the two angles are said to form a linear pair.

   The procedure followed in teaching the idea of supplementary angles may be followed to teach the idea of linear pair also.

13. **Bisector of an angle**
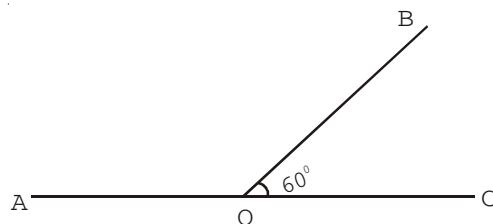
   A ray which divides an angle into two equal halves is called the bisector of the angle.

   The idea may be taught through paper folding. Take a neatly cut paper which forms a right angle at its vertices. Select any one vertex and fold the paper to form a crease which divides the angle formed at the vertex into two. Since the right angle formed at the vertex is now divided into two by the crease formed, to make angles of 45° on either side of it, the line formed by the crease formed is the angle bisector.

   The child must be explained to understand that bisection means dividing exactly into two halves. Also the difference between intersection and bisection - that intersection is just dividing and bisection denotes dividing exactly into two halves, should be described to the child.

14. **Adjacent angles**

   Two angles having a common arm and the same vertex are said to be adjacent angles.
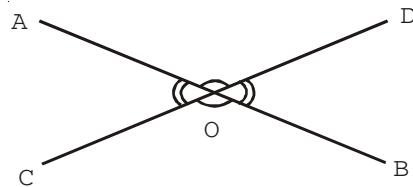
   Here OB is a common arm for the angles AOB and BOC, hence they are adjacent angles.

The procedure followed in teaching the concept of bisector of an angle is to be repeated as it is. Now the child may be allowed to explore the angle bisector, which enables the child to realize that the angle bisector facilitated the formation of two angles with a common arm, and hence such angles are called as adjacent angles.

15. Vertically opposite angles
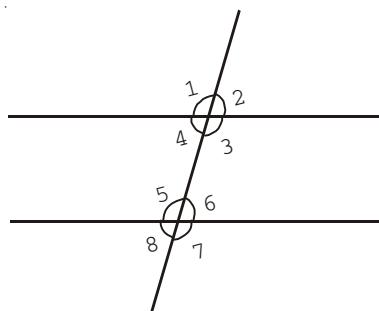
The angles formed by the intersection of two lines and which are not adjacent angles are called as the vertically opposite angles. In other words, when two lines intersect they form two pairs of angles. The angles which are opposite to each other are called as vertically opposite angles. Note that, the measurement of two vertically opposite angles is always equal.

Provide the child with a sheet of paper, which has been folded to form two intersecting lines in it. Facilitate the child to explore the nature of the four angles formed therein. Now explain to the child that the two pairs of vertically opposite angles formed by the intersection of any two lines will always be equal. The child may be facilitated to measure the equal angles which may enable him/her to understand that the vertically opposite angles are always equal.

16. Transversal

A line which divides two other lines at two different points is called as the transversal. If the two lines happen to be parallel to each other then the transversal forms a number of equal angles.

Three embossed lines, in which two of them are parallel to each other and the third one passing through the other two is to be prepared on a sheet of paper and can be given to the child to explore. The arrangement of the three lines may enable the child to conceptualize the idea of transversal. To reinforce the conceptualization of the idea, the child may also be explained about the nature of various angles formed in the transversal. The idea of transversal may be taught effectively using paper folding (refer to section on creative mathematics).

TRIANGLES
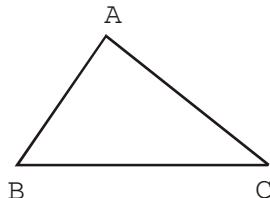
1. Triangle

A triangle is a closed figure formed by three line segments. In other words, a triangle is a plane figure with three straight sides and three angles.

A cut out of a triangle can be given to the child to explore its shape. On exploration, the child may clearly understand that a triangle has three sides and three angles.

Note : 1. Sum of the angles of a triangle is 180°.

2. Equilateral triangle, Isosceles triangle, Scalene triangle, Right angled triangle, Acute angled triangle, Obtuse angled triangle are the various types of triangles.

2. Parts of a triangle

A triangle has mainly six parts namely three sides and three angles. The meeting point of any two sides is called the vertex.

Provide the child with a triangle made out of paper or plastic. Allow the child to explore and then the parts of the triangle may be explained. In the process, different types of triangles may be given to the child to understand the nature of various types.

3. Types of triangles
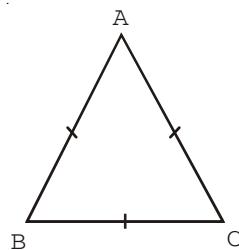
Triangle can be classified into three kinds according to their sides as scalene triangle, isosceles triangle and equilateral triangle.

According to angles, triangles are again classified into three kinds as acute angled triangle, right angled triangle and obtuse angled triangle.

The idea may be explained orally, in addition to the provision of different types of triangles constructed with paper.

4. Equilateral triangle

A triangle with all its three sides equal is said to be an equilateral triangle. Note that if the three sides are equal then each of the three angles will also be equal to 60°.
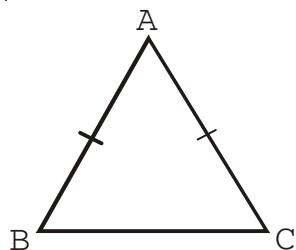
Explain the idea verbally. To enable the child to understand the idea better construct an equilateral triangle through paper folding and explain.

Related ideas such as sides opposite to equal angles are always equal and vice-versa etc., may also be explained simultaneously.

5. Isosceles triangle
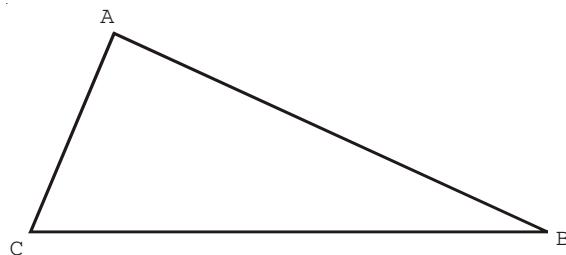
A triangle in which at least two sides are of the same length is called isosceles triangle. The idea may be explained orally supported by the provision of an isosceles triangle constructed with paper.

Related ideas like the side opposite to the greatest angle of the triangle is the longest side and vice-versa may also be explained simultaneously.

6. Scalene triangle
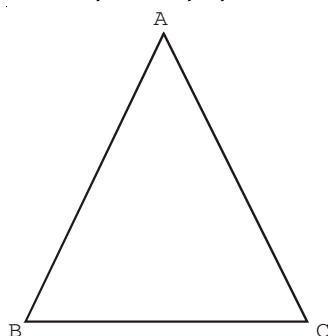
A triangle with no two sides are equal is called a scalene triangle. The idea may be explained orally supported by the provision of a scalene triangle constructed in paper.

7. Acute angled triangle
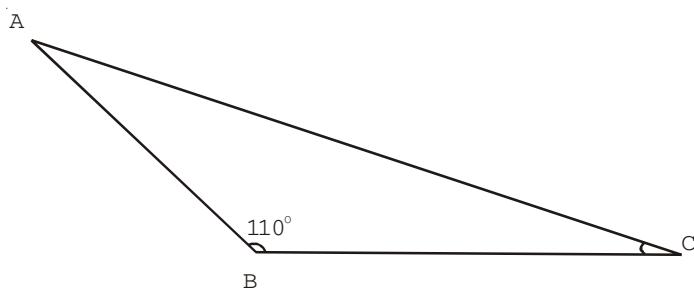
If all the three angles of a triangle are acute, that is if all the three angles are less than 90° then the triangle is called an acute angled triangle.

The idea may be explained orally supported by the provision of an acute angled triangle constructed with the help of a paper.

8. Obtuse angled triangle

A triangle in which one of its angles is greater than 90° is said to be an obtuse angled triangle.
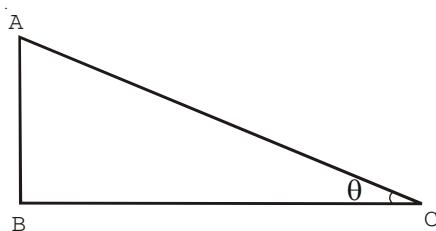
The idea may be explained orally supported by the provision of an acute angled triangle constructed with paper.

9.   Right angled triangle

A triangle with one of its angles equal to 90° is said to be a right angled triangle. In a right angled triangle, the side opposite to the right angle is called as the hypotenuse and the other two sides are termed as opposite side and adjacent side.
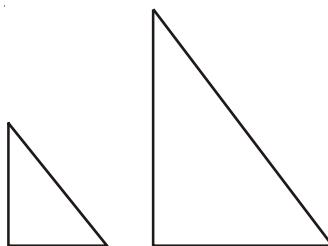
The idea may be explained orally supported by the provision of a right angled triangle constructed with paper. Follow the procedure mentioned in the creative mathematics section.

10.   Similar triangles

Two triangles are said to be similar if the measurements of their corresponding sides are proportional. Note that in similar triangles, the angles of one triangle will be equal to the angles of the other and the corresponding sides are in the same proportion.

The idea of similar triangles may be taught to the child with the provision of two triangles in which the measures of the corresponding sides are proportional. The child needs to be assisted to measure the corresponding sides of the two triangles and he/she must explained that the ratio between the corresponding sides are equal.

11. Equiangular triangles
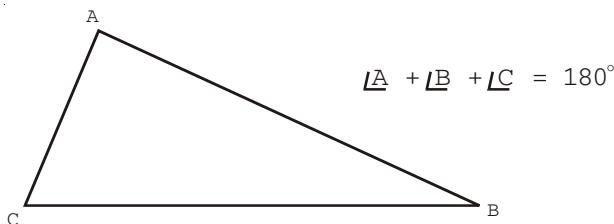
When the corresponding angles of two triangles are exactly the same they are said to be equiangular triangles. Note that the **equiangular triangles are similar triangles**.

The idea may be explained orally in addition to the provision of two equiangular triangles constructed either by paper or plastic.

12. The sum of the angles of a triangle is 180°

In any triangle, the sum of the measures of the angles of the triangle will always be equal to 180° or a straight angle.
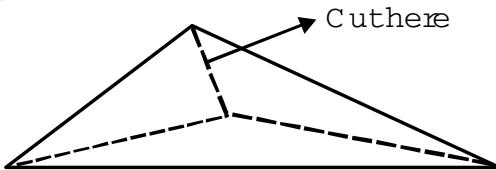
The procedure mentioned in the creative mathematics section may be followed to enable the child to understand the idea by performing the activity.

$$\angle A + \angle B + \angle C = 180°$$

13. In any triangle, the sum of the measures of any two sides is always greater than the third side

The idea can be demonstrated using paper folding or tactile material.

Explain the idea orally to the child. Now cut a triangle into three pieces so that the sides are intact.

Cut here
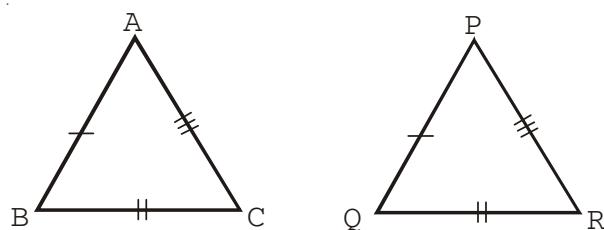
Select any two sides, place them together and measure the length of the combined sides. Now measure the third side. Allow the child to explore the lengths of both and let the child measure them. On measuring the sides, the child will be able to understand that the sum of the measures of any two sides of a triangle is always greater than the third side.

14. Congruency

Two objects identical in shape and size are said to be congruent. In geometry, congruency refers to two figures with the corresponding sides and angles identical. Also the difference between two-congruency and similarity must be explained to the child.

Two identical triangles in terms of corresponding angles and sides be given to the child and be asked to explore the magnitudes of their sides and angles. On verification, the child is able to conceptualize that the two triangles are identical in sides and angles and hence they are congruent.

Here the corresponding sides and angles of the two triangles are identical and hence $\triangle ABC \equiv \triangle PQR$
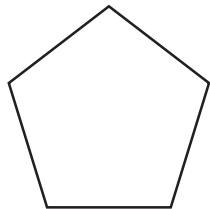
QUADRILATERALS

1. **Polygon**

A polygon is a plane figure formed by many line segments. The word 'poly' means many.

Square, rectangle, rhombus, pentagon, hexagon, etc., are all polygons. Provision of concrete polygons made out of thick boards and exploration may enable the child to understand the idea of polygons.

2. **Regular polygon**

If the sides and angles of a polygon are equal, then it is known as regular polygon. A polygon of five sides is called as a pentagon, a polygon of six sides is a hexagon and a polygon of eight sides is an octagon.

The child may be provided with a square and after exploration of its sides, the child may be able to understand that all four sides of the square are equal. Once the child is able to realize that the sides are equal, the idea that if all sides are equal then the geometrical figure may be termed as regular polygon is to be explained.
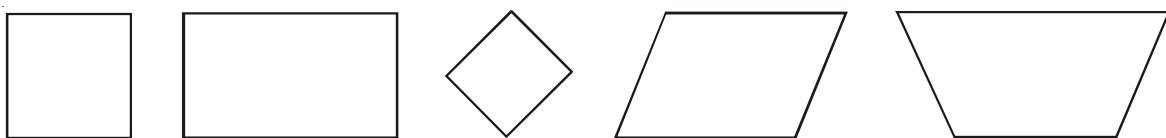
3. **Quadrilateral**

A quadrilateral is a closed geometrical figure on a plane formed by four line segments. In other words, any geometrical figure having **four straight sides** is called a

quadrilateral. Square, rectangle, rhombus, parallelogram and trapezium are some of the quadrilaterals. Shapes of these quadrilaterals can be given to the child and he/ she be asked to explore the shape of the same.
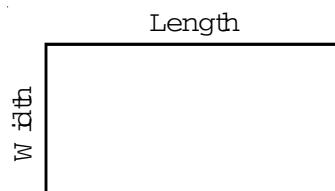
Related concepts such as, the sum of the angles in a quadrilateral is 360° can also be explained simultaneously.

In addition to oral explanation of the idea, provide the child with the different quadrilaterals made out of paper or plastic. On exploration, the child will be able to understand the nature of the various types of quadrilaterals and also understand the similarities and differences between each of them.

4. **Length/Width**

In a rectangle the longer side is always the length and the shorter side is the width. The child can be given a thick sheet of paper in the shape of a rectangle and he/she be asked to explore its shape. The child can observe tactually that there are two pairs of equal sides of which one pair is longer than the other. Now the child may be explained that the longer side is the length and the shorter side is the width of the rectangle. The Braille slate or abacus may also be useful to explain this concept.
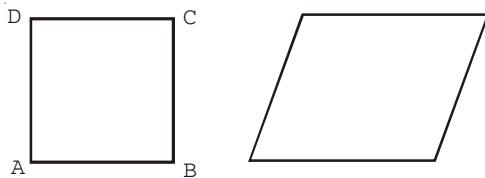
5. **Square**

In a quadrilateral if all the four sides are equal and if all the angles are right angles then the quadrilateral is called a square.

A cut out of the square may be given and the child be asked to explore the shape.
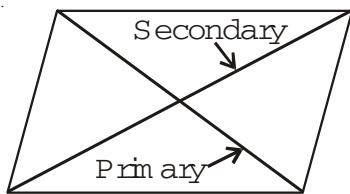
Note : 1. If all the sides are equal but not at right angles to each other, then it becomes a rhombus.

2. The area of square is (side × side) square units.

3. Perimeter of a square = 4(side) units.

6. Adjacent sides

In a quadrilateral the sides having the common vertex are termed as adjacent sides. When the child is provided with any quadrilateral either made of plastic or paper, he/she will be able to understand the idea through exploration supplemented by the description given by teachers.

7. Diagonal

The line joining the opposite vertices of a quadrilateral is called as the diagonal. Diagonals are of two types, primary diagonal and the secondary diagonal. Primary diagonal is the one which connects the top left vertex and the bottom right vertex. Secondary diagonal is the one which connects the bottom left vertex and the top right vertex.

A neatly cut paper which is in the shape of either a square or rectangle be given to the child. Facilitate the child to fold the paper in such a way that the opposite vertices in alternate sides are connected, which is the diagonal.

8. Trapezium

A Quadrilateral with a single pair of parallel sides is called as the trapezium. An embossed diagram of a trapezium be prepared and the child asked to explore its
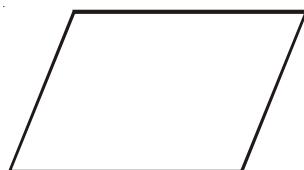
shape. Once the child conceptualizes the shape of a trapezium, relevant related concepts are to be explained. A paper which is cut in the form of a trapezium may also be provided to the child to explore its structure.

9.   Parallelogram

In a quadrilateral if the opposite sides are parallel to each other, then it is called a parallelogram.
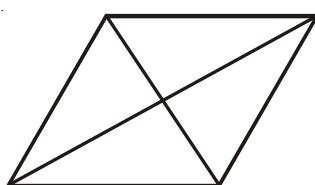
The idea may be explained orally supported by the provision of a concrete parallelogram constructed with a paper or plastic.

10. Rhombus

In a parallelogram if all the four sides are equal then it is called a rhombus. Note that in a rhombus all the sides are equal in length and the diagonals are perpendicular to each other. Let the child measure the diagonals and understand they are only perpendicular to each other and not equal in length whereas the diagnals in a square are of equal length.
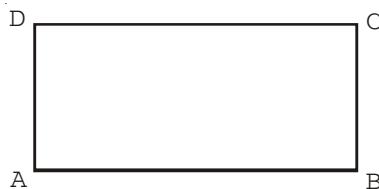
Provision of a rhombus made either of paper or plastic may enable the child to understand the idea better.

11. Rectangle

In a quadrilateral, if the opposite sides are equal and all the four angles are equal to 90° or a right angle then it is called a rectangle.

A hard board in the shape of a rectangle can be given to the child to explore. Otherwise, a neatly cut paper in the shape of a rectangle may be used to explain this concept.
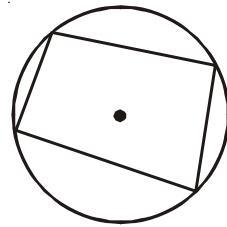
Note :   *1. In a rectangle the two pairs of opposite sides are equal. The longer side is the length and the shorter side is the width.*

         2. Area of rectangle = (length x width) square units

         3. Perimeter of a rectangle = 2 (length + width) units

12. Cyclic quadrilateral
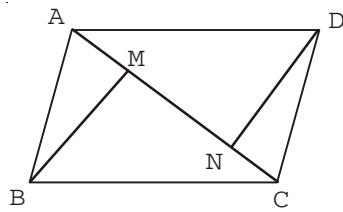
If all the four vertices of a quadrilateral lie on the circumference of a circle, then the quadrilateral is said to be a cyclic quadrilateral.

Verbal explanation of the idea along with the construction of a cyclic quadrilateral by the child as mentioned in the creative mathematics section will enable the child to understand the idea clearly.

13. Area of a quadrilateral

Any quadrilateral can be made into two triangles by its diagonals. The area of the quadrilateral can be found by adding the areas of the triangles by any such division.

Therefore, area of quadrilateral ABCD = Area of ABC + Area of ACD

$$= \frac{1}{2} \times d \times h_1 + \frac{1}{2} \times d \times h_2 \text{ square units}$$

$$= \frac{1}{2} d (h_1 + h_2) \text{ square units}$$
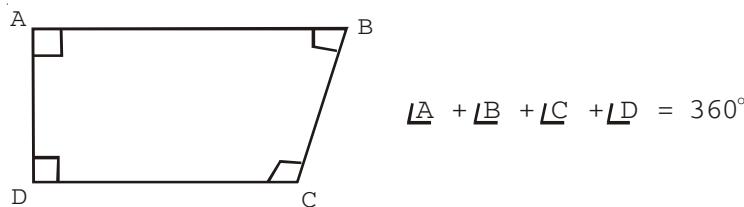
The idea may be explained orally with provision of adequate practice in memorizing the formula and facilitating the child to do the calculations on his/her own.

The idea may be taught using paper folding also.

14. Sum of the angles of a quadrilateral is 360º

The sum of the angles of a quadrilateral will always be equal to 360º. If the measure of an angle is equal to 360^0 it is also called as angle at a point.

The child may learn this idea by discovery. Construct a quadrilateral on a sheet of paper to form four angles of some measurement at its vertices. Cut the paper in such a way that the four angles formed in the vertices are taken apart. Enable the child to assemble the four papers in such a way that all the four pieces together form an angle of 360º. Thus the child may be able to understand that in any quadrilateral the sum of all the four angles formed will always be equal to 360^0.

$$\angle A + \angle B + \angle C + \angle D = 360^°$$

The Procedure is explained thoroughly in the Creative Mathematics section.

SIMPLE INTEREST

1. Principal

Principal is the sum of money lent or invested on which the interest is paid.

The concept of principal may be taught orally supported by necessary text material in Braille.

Enabling the child to the mock banking transactions may also help him/her understand the idea of principal and the related terminologies like interest and rate of interest etc.

2. Interest

The amount paid for use of the money lent is called the interest. The idea may be explained orally.

3. Rate of interest

The interest to be paid along with the principal, in terms of percentage for every year is called as the rate of interest and is denoted as 'I'.

4. Period

The time taken to return the money is called as the period and is normally denoted as 'n'. The idea may be explained to the child orally.

5. Simple interest

When a person is in need of money, he/she borrows from individuals or banks or finance institutions. At the time of returning the money, he/she must pay some additional money for the benefit of using the money borrowed for a specific period.

Principal is the sum of money borrowed.

Interest is the additional money to be given.

Amount to be returned = Principal + Interest

Eg. **:** *Find out the simple interest on a sum of 400 units for 1 year at the rate of 5% per annum.*

Principal   =   400 units

Rate of interest   =   5% per annum

Interest on 100 units for 1 year   =   5 units

Interest on 400 units for 1 year   = $\dfrac{5}{100} \times 400$

           =   20 units

The idea of simple interest may be explained with written examples.

Note : Unitary method
Unitary method is the process in which the value of the required is found by converting the given quantity into one unit.

Eg. **:** *Let the cost of 5 books be 50 units. To findout the cost of 10 books, first the cost of 1 book must be found.*

Cost of 5 books   =   50 units

Therefore, cost of 1 book $= \dfrac{50}{5} = 10 \, \text{units}$

Therefore, cost of 10 books = 10 × 10 = 100 units.

6. Formula for Simple Interest

Let, 'p' be the principal

   'n' be the number of years

   'r' be the rate of interest

Interest on 100 units for one year = r

Therefore, interest on 'p' units for one year

Interest on 'p' units for 'n' years $\quad \dfrac{pr}{100} \times n = \dfrac{pnr}{100}$

$$\text{Interest, } I = \dfrac{pnr}{100}$$

Once the child is clear with the idea of simple interest, then the formula for finding the simple interest may be taught. Initially, the symbols such as 'I, p, n, r' are to be made known and what they represent must also be emphasized upon.

7. **Recurring deposit**

A recurring deposit is a deposit in which the same amount is deposited for a particular period in regular intervals of time.

The idea may be explained to the child orally.

8. **Commission (Brokerage)**

When a house or a piece of land is purchased, some amount has to be given to the person who helps in the transaction. This amount is known as commission (or brokerage). The person who receives the commission is known as the broker and the commission is always given in terms of percentage.

The idea of commission may be explained to the child orally assisted by necessary text material in Braille.

9. **Discount**

Discount is a deduction from the usual cost of something. The idea of commission may be explained to the child orally.

10. **Calculating the number of days**

Calculation of the number of days is vital for certain organizations such as the bank wherein the exact number of days of lending a loan to a customer has to be calculated

so accurately for calculation of the interest. In calculation of the number of days in a specified period, the following points are to be taken into consideration.

- If given as, 'from this date' or 'to this date' then, both the days are to be included.

- If given as ' on' date, then the particular day is not to be included.

- If given as ' ending date' or ' inclusive' then the last day also must be included.

Eg. : *Find the number of days from 2nd January 2003 to 20th September of the same year.*

Month	Number of days
January	29
February	28
March	31
April	30
May	31
June	30
July	31
August	31
September	20
Total	**261**

The idea of calculating the number of days may be taught, when the child is comfortable with the number of days of each month of a year. This idea can be taught orally, assisted by necessary text material in Braille.

11. Finding the day of the week

If the day of a particular date is given, then the day of a later date can be found out. For this purpose the total number of days of the particular period is to be calculated first.

Eg. : If November 15,2003 falls on Saturday, find the day of December 31 of the same year.

Month	Number of days
November - Rest of the days	15
December - Prior to 31st	30
Total	**= 45 days**
	= 6 weeks + 3 days

Therefore the required day is 4 (3+1) days after Saturday, that is Wednesday.

Hence, December 31st of 2003 is Wednesday.

The idea may be taught orally to the child, and repeated exercise may develop his/her mental memory in performing this task.

Module 74

<u>SETS</u>

Set Language

George Ferdinand Ludwig Philip Cantor (1845-1918), a German mathematician developed the concept of set language.

1. **Set**

    A set is a collection of well defined distinct objects.

    Eg. : *A = {a, e, i, o, u}*

          B = {Cow, Goat, Hen}

    Provide the child with a few known objects, say for instance, a Braille slate, stylus and an abacus. These objects are well defined and can form the elements of the set, say "assistive devices for the blind". Hence, if 'C' denotes this set then,

          C = {Slate, Stylus, Abacus}

    Note : It is conventional to denote a set by an English alphabet in capitals.

2. **Element**

    An object of a set is called an element or a member or an individual of the set. Normally a set is denoted by an upper-case letter like A or B and an element of a set by a lower-case letter such as x or y.

    Eg.: *A = { x, y, z}*

    Here x, y, z are called as the elements of the set.

The idea of element may be explained to the child orally with adequate examples.

3. **Well Defined Set**

A set in which the **elements can be listed out** is said to be a well defined set.

Eg. : *A = { Pen , Pencil}*

Here, both the elements of the set A, namely, pen and pencil are writing devices and they can be listed out and hence they are elements of a well defined set.

The child could be taught the concept orally in addition to the provision of the elements which are well defined as in the case of pen and pencil.

4. **Not well defined set**

A set in which the elements cannot be listed out is said to be a set, not well defined.

Eg. : *B = {Intelligent pupils in Class X}*

Unless the expression 'Intelligent' is linked to a numerical value or some definite measure, the elements of the set cannot be listed, and hence the set is not well defined.

The child with visual impairment can be taught the concept orally. If necessary few other terms which are not well defined such as beautiful, soft, courteous, etc., can also be given and the child be assisted to understand the concept clearly.

5. **Equal Sets**

Two sets A and B are said to be equal if they have **exactly the same elements**, and is denoted as A = B.

Eg. : A = {1, 2, 3}

   B = {3, 2, 1}

To enable the child to understand the concept, two pairs of identical objects, say, a slate, stylus and abacus be given to explore. Explain the concept orally supported by the concrete materials to enable the child to understand the concept of equal sets.

6.   Equivalent Sets

Two sets A and B are said to be equivalent, if they have the **same number of elements,** provided that the elements are not identical, and is denoted as **A ↔ B.**

Eg. : A = {1, 2, 3}

B = {4, 5, 6}

To enable the child to understand the concept, provide with two pairs of elements same in number, but different in shape or size. For instance, provide the child with slate, stylus and abacus on one hand and pen, pencil and eraser on the other hand. By exploration, the child is able to realize that though the number of objects on both his/her hands are equal, but they are not the same objects. Sets of this nature are said to be equivalent sets.

Note : 1.   All equal sets are equivalent sets.
2.   But all equivalent sets are not equal sets.

7.   Cardinal number of a set

The number of elements present in a set is said to be its cardinal number. The cardinal number of a set having three elements is 3.

Eg. : A = {4, 6, 7}

Here, the set A contains three elements and hence the cardinal number of the set is 3, and is denoted as $n(A) = 3$

Explain the concept to the child orally and then provide him/her with a few well defined objects, say, pen and a pencil, which form a set of writing devices.

8.   Finite Set

A set   with countable number of elements is called a finite set.

Eg. : A = {a, b, c}

Here the set A contains countable number of elements, that is, three.

9. Infinite Set

A set with uncountable number of elements is said to be an infinite set. In other words, if the cardinal number of a set could not be found then the set is said to be an infinite set.

Eg. : *C = {1, 2, 3, …}*

Provide the child with a bowl full of sand grains and ask him/her to count the grains. Counting the grains is practically impossible, and now explain the concept of an infinite set, orally.

Note : *In other words, a set which is empty or consists of a definite number of elements is called finite, other wise the set is infinite.*

For example,

> *Set of days of the week*
>
> *Set of vowels in the English alphabet*
>
> A = $\{ x \mid 1 \leq x \leq 10, x \in N\}$
>
> B = $\{ x \mid 15 > x, x \ N\}$ *are all finite sets, while*
>
> N – *Set of natural numbers*
>
> Z – *Set of integers*
>
> Q – *Set of all rational numbers*
>
> C = $\{ x \mid x > 11, x \in N\}$

the set of points in a line, etc., are all infinite sets.

10. The Null Set / The Empty Set / The Void Set

A set with no elements is said to be the null set or the empty set or the void set.

Eg. : B = *{Horses with six legs}*

C = $\{ x \mid x+8 = 4, x \in N\}$

D = $\{ x \mid x+4 = 4, x \ N\}$

The concept of null set can be explained orally. If the child is not able to understand the concept still, provide the child with an empty box with no objects inside. If the box is treated as a set, then it is a null set with no elements.

11. Universal set

A set which contains all the elements are under consideration is said to be a universal set and is denoted as U.

Eg. : *If   A = {1, 2, 3}*

B = {3, 4, 5}

Then, one of the universal sets for the above sets A and B is

U = {1, 2, 3, 4, 5, 6}

A group of sets can have any number of universal sets, the only condition being that the universal set must contain all the elements which are under consideration.

To enhance the level of understanding, provide the child with a pencil, pen on one hand and a slate, stylus on the other hand. The child is able to conceptualize that on one hand he/she has writing devices for non-disabled pupils and on the other hand he/she is having writing devices for the blind. Thus, one of the universal sets for the objects which he/she is in possession can be the set of writing devices.

12. Roster Form (or Tabular form)

In roster form, all the elements of the set are listed and are separated by commas.

Consider the set of vowels in the English alphabet. If C denotes this set, then, C = {a, e, i, o, u}

This idea could be explained to the child orally, distinguishing the difference between the different forms of representing a set.

13. Set Builder Form

In set builder form, **some property of the elements of the set is stated.**

Considering the set of vowels in the English alphabet again, if B denotes this set, then in set builder form the set is denoted as,

B = {x| x is a vowel in English alphabet}

This concept also has to be explained to the child orally, focusing on the difference between the roster form and the set builder form. The set builder method is also called as descriptive method.

14. Subset

If A and B are any two sets, then B is a subset of A, if A contains all the elements which are contained in B.

Eg. : Let, A = {1, 2, 3, 4}

B = {2, 4}

Here A contains all the elements which are contained in B and so B is a subset of A (also can call A as the superset of B).

To explain the concept to the child, ask him/her to consider the students, both boys and girls of his/her class as a set. Of this set, ask him to think of boys as one element and the girls as another element. Here, both the groups of boys or girls are subsets of the set namely, the students in the class.

15. Power Set

A set which **contains all the subsets** of the given set is called the power set.

Eg. : Let   A = {1, 2}

Then the power set of A denoted as P(A) is given by,

Observe that if a set has 2 elements then its power set will be having 4 (=2^2) subsets. In general, if a set A has 'n' elements then its power set P(A) will be having 2^n elements.

The child with visual impairment can be assisted to understand this concept, with the provision of a set of objects and the grouping of the objects in different combinations. The only area that the child may have little difficulty is in the formation of a null set, which is impractical, and could very well be explained by the teacher orally.

The idea of null set may be tried out using a class room situation also. Suppose a school gets over at 4 p.m. A teacher wants to meet three students namely A,B and C after the final bell rings. Then the different possibilities that the teacher meeting the three students together, in groups or individually are listed as follows:

a.   A,B,C

b.   A,B

c.   B,C

d.   C,A

e.   A

f.   B

g.   C

h.   None, which is the null set.

The idea may be followed to teach the concept of power set also.

16. **Proper Subset**

For a set, the subsets other than the given set are called as proper subsets.

In the example given in item 15, {1}, {2} and { } are proper subsets for the given set.

17. **Improper Subset**

For a set, the **given set itself is said to be the improper subset.**

In the example given in item 15, the given set {1, 2} is the improper subset.

The idea of improper set can also be explained to the child orally, supported by tactile material depicting the sets.

18. Complement of a Set

For a set A, its complement denoted as A' (or A^c) is the set of elements of the universal set other than the elements of A.

Eg. :   If U   =   { 1, 2, 3, 4, …, 9}

A   =   {2, 4, 6, 8}

Then, A'   =   {1, 3, 5, 7, 9}

To explain the concept to the child, ask him/her to consider the students of his/her class as a set. Therefore the universal set in this case is the set containing both boys and girls of the particular class. If A denotes the set of boys in the class then the complement of A is the set of girls in the same class.

19. Cardinal number of the power set of a set

The cardinal number of the power set of a set is found using the formula, if n(A) = n, then the number of elements in the power set of A i.e., $n[P(A)] = 2^n$.

For instance, consider the set A = {1, 2, 3}. Here, n(A) = 3. Then $n[P(A)] = 2^3 = 8$

Provide three objects (abacus, stylus and slate) to the child and ask him/her to group the objects in various ways. The combinations are as follows :

C1   :   {abacus, stylus, slate}
C2   :   {abacus},
C3   :   {stylus}
C4   :   {slate}
C5   :   {abacus, stylus}
C6   :   {abacus, slate}
C7   :   {stylus, slate}
C8   :   {   }

20. Singleton Set

A set with only **one element** is said to be a singleton set.

Eg. : *B = {4}*

Here, n(B) = 1, and hence B is a singleton set.

21. Union of Sets

If A and B are any two sets, then their union denoted as A $\cup$ B, is the set of elements belonging either to A, or to B, or to both.

Eg. :   If A   = {1, 2, 3} and

             B   = {3, 4, 5}

    Then, A$\cup$B   = {1, 2, 3, 4, 5}

Note that even though one particular element, namely 3, appears in both the sets, in the union set it has to be written only once. In general, a set should never contain an element more than once. In other words, a set should contain **well defined distinct objects only**.

A number of concepts can be taught by using the fingers of the hand. Take a particular hand for demonstration. Consider the union of two sets with the fingers as the sets, say for instance, {little finger, ring finger, middle finger} and {little finger, index finger, thumb} as two different sets. The union of these two sets is the set with all the five fingers of the hand. Note that the hand is not having two little fingers and hence the union set will contain only one little finger, even though it is repeated in both the sets.

 The ideas of intersection, complementation and difference can also be taught using the fingers of the hand.

22. Intersection of Sets

If A and B are any two sets, then the intersection of these two sets denoted as A ∩ B is the set of elements which are common to both the sets A and B.

Eg. :   If A   = {1, 2, 3, 4}

             B   = {2, 3, 5}

Then, A ∩ B = { 2, 3}

To enable the child to understand the concept, provide two sets of objects such that there are some objects common in both the sets, say for instance, a pen, pencil, eraser in one hand and a pencil, scale, stylus on the other hand. On exploration, the child will be able to realize that, one object namely, the pencil is common in both the sets and hence the intersection of two sets is the pencil.

23. Overlapping Sets

Two sets are said to be overlapping if there exists at least one element common in both the sets. Note that overlapping/non-overlapping is just a literary description and has no definite symbols.

Eg.: If   A = {3, 4, 5}

          B = {4, 6, 8}

In the above two sets, the element 4 is common in both the sets and hence the sets A and B are overlapping sets.

24. Non-Overlapping Sets (Disjoint Sets)

Two sets are said to be non overlapping if there exists no common elements among the two sets.

Eg. : Let A = { 1,2,3}

          B = {4, 5, 6}

In the above two sets, as no element is common and hence the sets A and B are non-overlapping sets.

To enable the child to understand the concept, provide two different sets of objects, say, pen, pencil, eraser in one hand and stylus, scale, abacus on the other hand. Now ask the child to explore the objects in his hands and the child will be able to realize that among the two sets there exists no common object, and hence the two sets are non- overlapping sets.

25. Set difference

If A and B are any two sets, then the set of all elements of A which are not in B is called the difference set which is denoted as **A-B**. Note that A-B and B-A are not equal.

Eg. : If A = *{1, 2, 3}*

  B = *{3, 4, 5}*

Then A - B = *{1, 2}*

Also, B – A = *{4, 5}*

The idea may be explained to the child orally supported by relevant text material in Braille. If the child is still not clear with the idea, the methodology followed in teaching the concepts of union and intersection of sets, with the same objects may be followed.

26. Symmetric difference

The symmetric difference of two sets A and B is the union of their difference sets (A-B) and (B-A), denoted as the set (A-B) $\cup$ (B-A). The symmetric difference of two sets A and B is denoted as A$\triangle$B and is read as A delta B.

i.e., A$\triangle$B = (A-B) $\cup$ (B-A)

Eg. :   *If A*   =   *{1, 2, 3}*

             B   =   *{3, 4, 5}*

    Then A - B   =   *{1, 2}*

    Also, B – A   =   *{4, 5}*

      $\therefore A \triangle B$ =   (A-B) $\cup$ (B-A)

                 =   *{1, 2, 4, 5}*

27. Cartesian product of two sets

If A and B are two sets, a set of ordered pairs can be **formed associating every element of A as the first component and every element of B as the second component**. The set of such ordered pairs is called the Cartesian product or Cross product and is denoted as A×B.

Eg. :   *Let A*   =   *{1, 2}*

             B   =   *{3,4}*

    Then, A × B   =   *{ (1,3), (1,4), (2,3), (2,4) }*

Note : In general, if n(A) = p and n(B) = q, then n(A × B) = pq

28. An identity in set language

If A and B are any two non-empty overlapping sets, then

n(A $\cup$ B) = n(A) + n(B) – n(A $\cap$ B)

Note that if A and B are non-overlapping(disjoint) sets, then n(A $\cup$ B) = n(A) + n(B)

The methods used in the previous examples may also be adopted in the case of identity in set language.
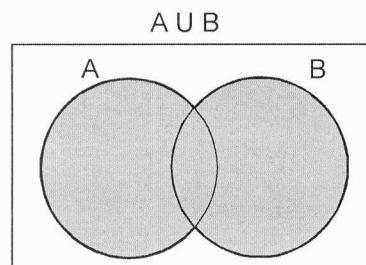
VENN DIAGRAM

Venn diagram
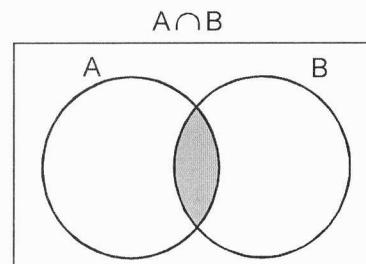
John Venn, an English mathematician introduced the idea of representing the sets in the form of diagrams, and hence the idea was named after him. Conventionally, the universal set is denoted by a rectangle and the sets within the universal set by circles.

Tactile diagrams depicting the following shaps may be prepared to teach various concepts using Venn diagram

1. $A \cup B$

$A \cup B$

2. $A \cap B$

$A \cap B$

3. A^c

A^c

4. B^c

$$B^c$$

5. (A $\cup$ B)^c

$$(A \cup B)^c$$

6. (A $\cap$ B)^c

$$(A \cap B)^c$$

7. A - B

$$A - B$$

8. B - A
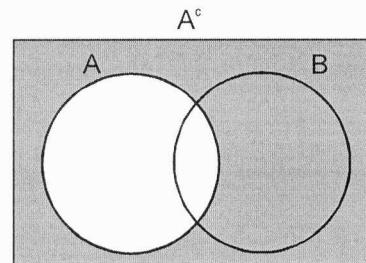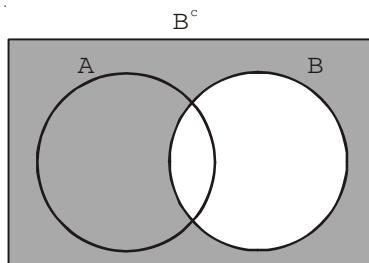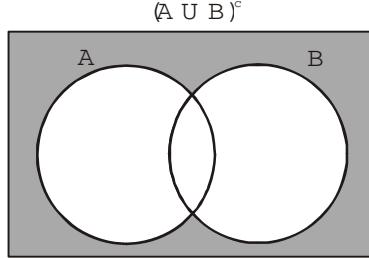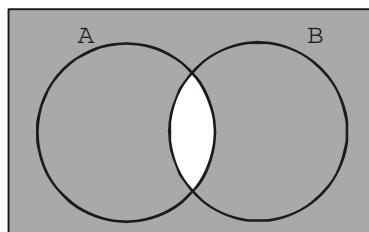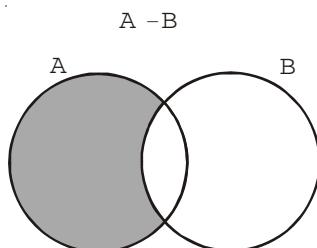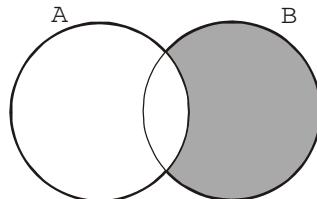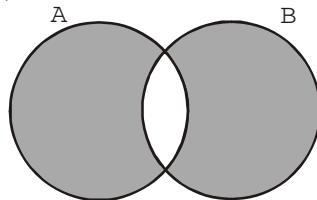
$$B - A$$

9. $A \triangle B = (A-B) \cup (B-A)$ $(A-B) \cup (B-A)$

De Morgan's laws on complementation

10. The complement of the union of any two sets is equal to the intersection of their complements.

$$(A \cup B)' = A' \cap B'$$

11. The complement of the intersection of any two sets is equal to the union of their complements.

$$(A \cap B)' = A' \cup B'$$

De Morgan's laws on set difference

12. $A - (B \cup C) = (A - B) \cap (A - C)$

13. $A - (B \cap C) = (A - B) \cup (A - C)$

The idea of Venn diagram and the different laws using the Venn diagram can be taught with the assistance of embossed diagrams of all the set notations used.

RELATIONS

1. Relation

A relation is a correspondence between the elements of two sets A and B. A relation is denoted by the letter R. If x is a member of A and y is a member of B and if x is related to y then the relation is denoted as x R y.

Eg. :   Let A = {2, 5} and B = {4, 6, 8}

The carterian product A x B = {(2, 4) (2, 6) (2, 8) (5, 4) (5, 6) (5, 8)}

If R denotes the relation "is less than" then 2R4 (since 2<4), 2R6, 2R8, 5R6 and 5R8

$\Rightarrow$ R = { (2, 4), (2, 6), (2, 8), (5, 6), (5, 8) }

Thus the set of ordered pairs denote a relation R from A to B.

The idea may be explained orally. Further, the concept of relation using two sets may be prepared as an embossed diagram and be given to the child to explore.

2. Reflexive relation

A relation on a set A is said to be reflexive if every element of A is related to itself. In other words, for all a $\in$ A, a R a.

Eg. : In a set of triangles, the relation "is congruent to" is reflexive as every triangle is congruent to itself.

The idea may be explained orally in addition to the provision of necessary Braille text material.

3. Symmetric relation

A relation R, on a set A is said to be symmetric, if aRb and bRa, for all a, b $\in$ A.

In symbols, aRb implies bRa

Eg. **:** In the set of all lines let R denote the relation "is parallel to". If a line l_1 is parallel to another line l_2, then l_2 is parallel to l_1. Hence $l_1 Rl_2 \Rightarrow l_2 Rl_1$. So R is symmetric.

4. Transitive relation

A relation R on a set A is said to be transitive if aRb, bRc imply aRc for all a,b,c $\in$ A.

In symbols, aRb, bRc imply aRc

Eg. **:** Consider the set N of natural numbers. Let a, b, c $\in$ N. Let r denote the relation "is less than". R is transitive since a < b and b < c $\Rightarrow$ a < c.

5. Equivalence relation

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric and transitive. In other words, a relation in a set A is equivalence relation if the following conditions are satisfied.

a R a, for all a $\in$ A

a R b implies b R a, for all a, b $\in$ A

a R b and b R c imply a R c, for all a, b, c $\in$ A

Examples from family may also used.

Assume x, y, and z are brothers from the same family A. X$\in$A, XRX, XRY implies YRZ XRY and YRZ imply XRZ for all X, Y, Z $\in$ A

Eg. **:** In the set of lines on a plane, the relation "is parallel to" is reflexive, symmetric and transitive. Therefore the relation "is parallel to" is an equivalence relation.
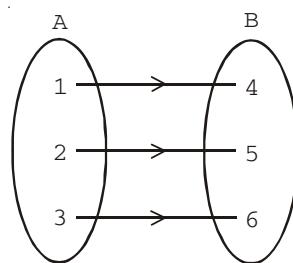
FUNCTIONS

1. Function

Let A and B be any two non-empty sets. Let f denote some rule which associates with every element of A, a unique element of B. Then, f is a function or a mapping from A to B and is denoted as f : A→B.

Note that a relation is a function if there is one and only one image for each element in the domain.

Two embossed sets with some elements in each may be prepared on a sheet of paper and be used for teaching the idea of functions and its types.

Eg. :
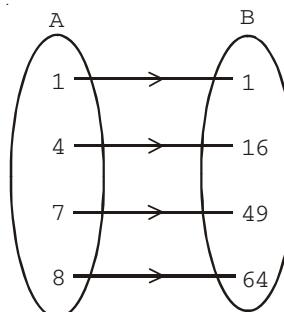
f : A→B and f(x) = x+3

2. Image and pre-image

Let f : A→B, a∈A. If a is mapped to b ∈ B, then the image of a is b. If b is the image then a is its pre-image.

Eg. :

f : A⟶B and f(x) = x²

Here 1, 4, 7 and 8 are pre-images and 1, 16 49, 64 are images
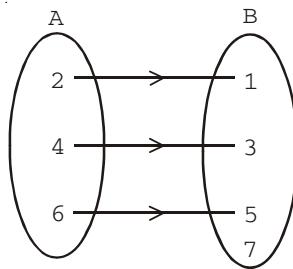
Verbal explanation assisted by two sets prepared in Braille may help the child to understand the ideas of image and pre-image.

3. **Domain and Co-domain**

Let f : A⟶B be a function. Then A is called the domain and B is called the co-domain of the function f.

The idea may be taught orally with necessary text material in Braille.

Eg. :

f : A⟶B and f(x) = x-1

Here, domain   = {2, 4, 6}
     Co-domain   = {1, 3, 5, 7}

4. **Range**

The set of elements in the co-domain, (which are images) form a subset of the co-domain and is called the range of 'f'.

Eg. : In the above example, range = {1, 3, 5}

Note:

1. *If an element of the domain has more than one image in the co-domain then it is not a function.*

2. *More than one element of the domain may be mapped to the same element in the co-domain.*

3.   *Each and every element of the domain should have an image in the co-domain, but all the elements in the co-domain need not have pre-images.*

4.   *An element in the co-domain may have more than one pre-image.*

The idea may be taught orally with the assistance of a tactile diagram.
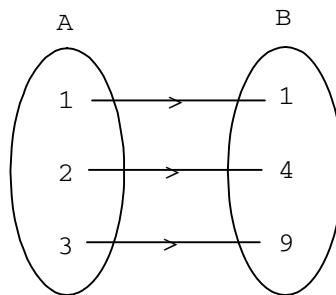
> A multipurpose magnetic board with oval shapes, pointers and embossed braille numbers will be useful to teach various concepts in set algebra. As movement of shapes and arrows are possible on magnetic board, such a device may be useful for teaching purposes.

5.   One-to-one function

The function $f : A \rightarrow B$ is said to be one-to-one if different elements in A have different images.

In other words, one-to-one function means that to every element of the domain A, there exists a unique image in the co-domain B.

Eg. :

$f : A \rightarrow B$ and $f(x) = x^2$
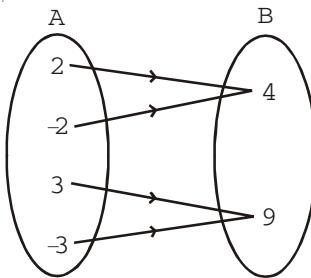
6.   Many to one function

The mapping $f : A \rightarrow B$ is called many to one, if two or more elements of set A, correspond to one element of set B.

In other words, when two or more elements of the domain A, correspond to the same element of the co-domain B, that is an element of set A, then it is called as many-to-one function.
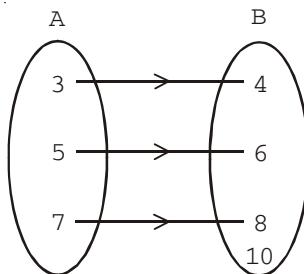
Eg. :

f : A→B and f(x) = x²

7.   Into function

The mapping f : A→B is called into, if there is at least one element of set B which has no pre-image in set A.
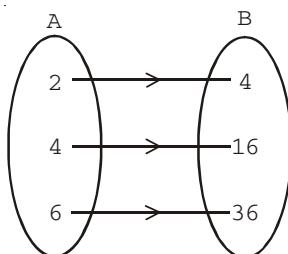
f : A→B and f(x) = x+1

Note that 10 has no pre-image in domain A.

8.   Onto function

The mapping f : A→B is called onto if every element in set B has pre image in set A. That is, a function f : A→B is called onto if its range is equal to its co-domain B.
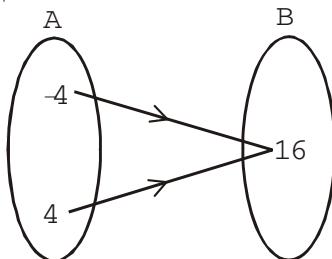
f : A→B and f(x) = x²

Tactile diagram may be prepared to teach this concept.

ICEVI

9. **Constant function**

A function f : A→B is called a constant function if every element of A has the same image in B. In other words, f associates the same element b ∈ B with each element at a ∈ A.
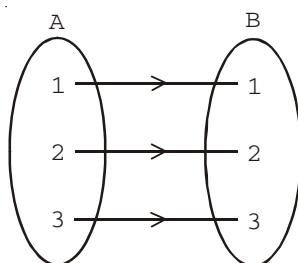
f : A→B and f(x) = x²

10. **Identity function**

Let A be a non-empty set. A function f : A→A is called an identity function if each element of A is associated with itself under 'f.' In other words, 'f' is an identity function of A if f(x) = x for each element x ∈ A.

Two sets which are tactually attractive with some elements on both may be prepared and used for explaining all the types of functions.

Eg. :

f : A→B and f(x) = x

11. **Ordered pairs**

Any function f : A→B can be represented as a set of ordered pairs as follows:
f = {(x, y); x ∈ A, y ∈ B}

As the child is already familiar with the idea of ordered pairs, representation of a function in the form of an ordered pair may be easy for the child to understand.

Consider A = {1, 2, 3}

B = {1, 8, 27}

$f(x) = x^3$
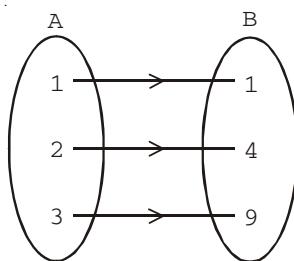
In ordered pairs, f = { (1,1), (2,8), (3,27) }

12. Arrow diagram

A function can be represented as an arrow diagram also, in which the two sets A and B are represented by two closed curves in which the elements of both the sets are enclosed.

Two embossed diagrams depicting two sets of a function may be prepared and the child be given to explore. Mapping of every element of the domain to the corresponding element of the range may be shown with the aid of a small wooden strip or a string.

Eg. :
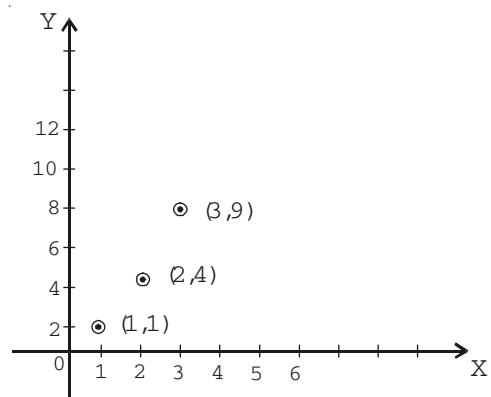
f : A→B and $f(x) = x^2$

13. Graph

Draw the X and Y axes on a paper and plot the points (x, y) for every x ∈ A and y ∈ B under f. The set of points that are marked on the plane is called the graph of the function f.

The idea may be explained with the assistance of tactile rectangular coordinate system prepared on a sheet of paper.

Braille lines may also be prepared to teach the concept of graph. A geo-board with rubber bands are also useful to teach graphs effectively.

BINARY SYSTEM

1.   Binary system

The system which uses only two digits namely 0 and 1 is called the binary system. The system which uses ten digits namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is called as the denary or decimal system and the system which uses eight digits namely 0, 1, 2, 3, 4, 5, 6, 7 is called as the octal system. There are some other systems also such as the hexadecimal system which uses 16 digits. All the present day computers make use of the binary system, thus transforming all the data into the computers through the machine language which uses only two digits, namely 0 and 1.

2.   Conversion from denary to binary

A denary number can be converted into a binary number by dividing the given number by 2 continuously until the remainder is obtained as either 0 or 1. The remainders which are obtained in each step, form the binary equivalent of the given denary number.

Eg. : Express 15 as a binary number

$$
\begin{array}{c|l}
2 & 15 \\
\hline
2 & 7 \text{ -1} \\
\hline
2 & 3 \text{ -1} \\
\hline
  & 1 \text{ -1} \\
\end{array}
$$

$\therefore$ $15 = 1\ 111_2$

The magnetic board with embossed braille symbols and written material in braille depicting the visual layout of the division process will be useful to teach the concept.

3. Conversion from denary to octal

A denary number can be converted into an octal number by dividing the given number by 8 continuously until the final remainder is either equal to 0 or a value less than 8, and the remainders in each step form the octal equivalent of the given denary number. The idea can be taught with an example with the assistance of necessary Braille text material. The child needs to be given adequate practice in performing the sums on his/her own.

Eg. : Convert 123 into an octal number

```
8 | 123
8 | 15 - 3
  | 1 - 7
```

$\therefore$ 123 = 173$_8$

4. Binary addition

The following rule is to be followed in adding two or more binary numbers.

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 10

10 + 1 = 11

11 + 1 = 100

100 + 1 = 101

101 + 1 = 110

110 + 1 = 111

111 + 1 = 1000

Eg. : Add : 1101 + 1111

```
  1101 +
  1111
 ─────
 11100
```

This is a simple logic. When the numbers are added, the added value should be the next number involving 0 or 1. For example, in 10+1, the immediate next number is 11. In 11+1, the next available number involving 0 or 1 is 100. The child may be helped to understand this logic through repeated exercies.

In binary operation two numbers should be added at a time. In this addition, the operation should start from right to left. 1 + 1 = 10, and therefore put 0 as the added value and carry over 1 to the column in the immediate left. Now 1 + 0 = 1, and 1 + 1 = 10, put 0 in the result column and the 1 should be carried over to the immediate left column and so on.

5.   Binary subtraction

As in the case of binary addition, in binary subtraction also the following rules are to be followed.

$$0 - 0 = 0$$
$$1 - 0 = 1$$
$$1 - 1 = 0$$
$$10 - 1 = 1$$
$$11 - 1 = 10$$
$$100 - 1 = 11$$
$$101 - 1 = 100$$
$$110 - 1 = 101$$
$$111 - 1 = 110$$

The same logic applies in subtraction too. For example, 10 – 1 = 9 but the next lowest binary number is 1 and therefore, 10 –1 = 1. However, 11 – 1 = 10 which is already in the form of a binary number. Similarly 1000 – 1 = 111. since 111 is the next available number with binary coding. The child should not confuse the binary addition and subtraction with the regular addition or subtraction.

Eg. : Subtract 1111 – 1011

```
  1111
  1011
  ────
   100
```

6.   Binary multiplication

As the binary system involves only two digits 0 and 1, the entire multiplication process is complete with the knowledge of the product of these two digits in different combinations. Once the digits are multiplied the rest is only addition of the two or more digits involving 0 and 1.

The basic rules regarding multiplication in binary system are,

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Multiplication table

×	0	1
0	0	0
1	1	1

If the child is aware of these rules, then the process of multiplication involves only addition which the child is already aware of.

Eg. : Multiply 101×110

$$
\begin{array}{r}
101 \\
110 \\
\hline
\mathbf{000} \\
101 \\
101 \\
\hline
\mathbf{11110} \\
\hline
\end{array}
$$

7. Binary division

The process of binary division may be taught once the child is aware of the basic rules.

$$0 \div 1 = 0$$
$$1 \div 1 = 1$$

As the rules of binary division are similar to that of normal division the child may be comfortable in performing division in binary system.

Eg. : $1101 \div 10$

$$
\begin{array}{r}
10 \overline{)\ 1101}\ (11 \\
\underline{10} \\
10 \\
\underline{10} \\
01 \\
\hline
\end{array}
$$

Quotient : 11

Remainder : 1

MATRICES

1. Matrix

A matrix is an arrangement of numbers in rows and columns. In a matrix the numbers arranged horizontally constitute the rows and the numbers arranged vertically constitute the columns.

Eg. : $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ 7 & -8 & 9 \end{pmatrix}$

The concept of matrices could be taught to the child by either using the abacus or through situational approach, taking into account the seating arrangement of the students in the class. The beads of the lower abacus form a matrix with 4 rows and either 13 or 15 columns. The seating position of the students in the class if arranged in an uniform pattern as rows and columns can be cited as a concrete example for a matrix.

Note that fractions and other operations in algebra may be presented either in the linear way or in the spatial way, but the matrix concept can only be presented in the spatial way and therefore, use of a magnetic or a Flannel Board may be necessary. The Braille numbers may be placed on the board in the spatial manner for the child to explore. The brackets of the matrices may also be shown on the board so that the child gets an idea of how the matrix is presented in the visual form.

Note : General form of a matrix.

If A is a matrix of order m × n, then,

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix} \quad m \times n$$

Note : LEADING DIAGONAL

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

In the above matrix the elements of the leading diagonal are a_{11}, a_{22}, a_{33}.

2. **Order of a Matrix**

The order of a matrix is given by the number of rows into the number of columns of the matrix. For example, if a matrix has 3 rows and 4 columns then the order of the matrix is denoted as 3×4. Note that the sign '×' between the rows and columns does not denote multiplication. Also note that a matrix with 3 rows and 4 columns will contain $3 \times 4 = 12$ elements. In general, if a matrix has 'm' rows and 'n' columns then the matrix will consist of 'mn' elements. The elements of a matrix are also called as entries of the matrix.

Eg. : $A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & -3 & -4 \end{pmatrix}$

Here the matrix A has 2 rows and 3 columns and hence the order of the matrix is 2×3. Also note that the total number of elements in the matrix is $2 \times 3 = 6$.

Once the child is clear with the concept of matrix, the method of finding the number of elements in a matrix can be taught with the help of magnetic board where rows and columns are arranged.

3. **Square Matrix**

A matrix with equal number of rows and columns is said to be a square matrix.

Eg. : $B = \begin{pmatrix} 2 & -4 \\ 3 & -6 \end{pmatrix}$

Here, the matrix B has two rows and two columns i.e., equal number of rows and columns and hence the matrix B is a square matrix of order 2×2.

The concept of square matrix can be taught to the child with examples, supported by a situational approach, wherein students, say in three rows containing three members in each row. The child may be asked to explore the arrangement of the students which may enable him to understand the concept.

4. **Rectangular Matrix**

A matrix with unequal number of rows and columns is said to be a rectangular matrix.

Eg. : $C = \begin{pmatrix} -1 & -2 & 3 \\ -2 & -4 & 9 \end{pmatrix}$

In this example, the matrix C contains 2 rows and 3 columns i.e., unequal number of rows and columns and hence C is a rectangular matrix.

Note : All square matrices are rectangular matrices, but the converse is not true.

Situational approach can be adopted to enable the child to understand this concept in addition to verbal explanation, as in the case of a square matrix. To enhance the learning of the child, a concrete object, say, the abacus itself can be given to explore the lower abacus which has 4 beads in each column and the number of rows may differ depending on the abacus. If the abacus is having 15 columns then the lower abacus forms a matrix of order 4×15 and if it contains 13 columns then the abacus forms a matrix of order 4×13. The Flannel or Magnetic board may also be used.

5.   Row Matrix

A matrix with a single row is said to be a row matrix.

Eg : $B = \begin{pmatrix} 1 & -2 & -3 \end{pmatrix}$

The matrix B has only one row and three columns and hence it is a new matrix of order 1×3.

The seating arrangement of students of a particular desk in the class can be treated as a concrete example for enabling the child to understand the idea.

The upper abacus which contains a single row and 13 or 15 columns, as the case may be, can be used as a concrete example for teaching the idea of row matrix.

6.   Column Matrix

A matrix with a single column is said to be a column matrix.

Eg : D - $\begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}$

Here, the matrix D has 3 rows and only a single column and hence D is a column matrix of order 3×1.

The child may be asked to explore a particular column of the abacus which contain 4 rows and a single column, thus making a column matrix.

The students standing in a line also becomes a column matrix.

In the expression 1×3 matrix, '1' represents the 'row' and '3' represents the columns. The child should be oriented that the first number in the order always represents the row and the second number represents the column.

7. **Zero Matrix / Null Matrix**

A matrix with all its entries as zero is said to be a null matrix. Note that a null matrix can be of any order.

Eg : $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Here the matrix A has all its entries as zero, and also has two rows and three columns and hence A is a zero matrix of order 2×3.

As the child is already familiar with all the basic ideas of a matrix, the idea of a zero matrix can be taught to him/her orally supported by relevant text material in Braille.

8. **Diagonal matrix**

A square matrix with all its entries zero, except those along the leading diagonal is said to be a diagonal matrix. Note that only a **square matrix can be diagonal matrix**.

Eg. : $A = \begin{pmatrix} 2 & 0 \\ 0 & -7 \end{pmatrix}$

Here, the matrix A is a square matrix of order 2, and the matrix has all its entries zero, except those along the leading diagonal.

9. **Scalar Matrix**

A diagonal matrix in which all the diagonal elements are same is said to be a scalar matrix. Note that, a scalar matrix is basically a square matrix.

Eg : $C = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

Note that the above matrix C is a diagonal matrix in which all the diagonal elements are same, and hence the matrix is a scalar matrix.

The concept of scalar matrix can also be taught to the child verbally supported by relevant text material in Braille.

10. Unit Matrix / Identity Matrix

A diagonal matrix in which all the entries along the leading diagonal are equal to 1 is said to be a unit matrix or identity matrix. Note that an identity matrix has to be denoted only by the letter I. A unit matrix of order 2 is denoted as I_2 and a unit matrix of order 3 is denoted as I_3.

$$\text{Eg } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Here, the matrix I is an identity matrix of order 2 and hence it is denoted as I_2.

The knowledge of the child with regard to the basic ideas of matrices, may facilitate to understand the concept of unit matrix also, when the idea is narrated, supported by necessary reading material in Braille.

11. Negative of a Matrix

For a matrix A, its negative denoted as -A, is the matrix obtained by interchanging the signs of the entries of the matrix A.

Eg. : If A $= \begin{pmatrix} 2 & 3 & -4 \\ 5 & -2 & 5 \end{pmatrix}$

then, -A $= \begin{pmatrix} -2 & -3 & 4 \\ -5 & 2 & -5 \end{pmatrix}$

The idea can be explained to the child through spatial presentation of the matrix.

12. Addition of Matrices

Two matrices A and B can be added, only if they are of the same order. The resultant matrix is obtained by adding the corresponding entries of the two matrices.

Eg. : Let A $= \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and B $= \begin{pmatrix} -3 & 4 \\ 5 & 6 \end{pmatrix}$

Then, A+B $= \begin{pmatrix} 1-3 & 2+4 \\ 3+5 & -4+6 \end{pmatrix}$

$= \begin{pmatrix} -2 & 6 \\ 8 & 2 \end{pmatrix}$

Note that A + B = B + A. That is, matrix addition is commutative. If we are adding three matrices together then we can observe that matrix addition is associative also.

The idea could be taught to the child through the magnetic board supported by necessary reading material in Braille.

13. Subtraction of Matrices

Subtraction of two matrices is possible only if the matrices are of the same order.

Eg. : If A $= \begin{pmatrix} 3 & 5 \\ 4 & -6 \end{pmatrix}$ and B $= \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$

Then, A − B $= \begin{pmatrix} 3-1 & 5-3 \\ 4-2 & -6-5 \end{pmatrix}$

$= \begin{pmatrix} 2 & 2 \\ 2 & -11 \end{pmatrix}$

Note that A − B $\neq$ B − A. Hence **commutative property does not hold good** with respect to matrix subtraction.

14. Multiplication of Matrices

Two matrices A and B can be multiplied together, if the number of columns of the first matrix A is equal to the number of rows of the second matrix B.

If the order of matrix A is 2×3 and order of matrix B is 3×4 then A and B are said to be compatible for multiplication and the order of the product matrix A×B will be 2×4. In general, if A is a matrix of order m×p and B is a matrix of order p×n, then the product matrix A×B will be of order m×n.

Knowledge of all the basic ideas in matrices, may enable the child to understand the concept of matrix multiplication also when it is taught with examples.

Eg. : Let, $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 2 & 0 \\ -2 & 3 & 1 \end{pmatrix}$

Here the order of matrix A is 2×2 and the order of the matrix B is 2×3. That is the number of columns of the first matrix is equal to the number of rows of the second matrix. Hence the product of A and B is possible.

In finding the product of any two matrices, the elements of the first row of the first matrix are multiplied and added with the elements of the first column of the second matrix. Then the elements of first row of first matrix are multiplied and added with elements of second column of the second matrix and so on.

Therefore, the product of A and B, that is

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 4 & 2 & 0 \\ -2 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1\times 4)+(3\times -2) & 1\times 2+3\times 3 & 1\times 0+3\times 1 \\ 2\times 4+(-1)\times(-2) & 2\times 2+(-1)\times 3 & 2\times 0+(-1)\times(1) \end{pmatrix}$$

$$= \begin{pmatrix} 4-6 & 2+9 & 0+3 \\ 8+2 & 4-3 & 0-1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 11 & 3 \\ 10 & 1 & -1 \end{pmatrix}$$

Note that the product of the two given matrices AB is not equal to BA. That is matrix multiplication is not commutative. Matrix multiplication is commutative only when the two given matrices are equal.

15. Transpose of a Matrix

If A is the given matrix then its transpose denoted as A^T is the matrix obtained by interchanging the rows and columns of A. That is, to get the transpose of A, the rows of the matrix A are to be interchanged as columns and the columns are to be interchanged as rows.

If the order of matrix A is 3×4, then the order of the transpose matrix denoted as A^T is 4×3.

Note that in the case of a square matrix the order of a particular matrix and the order of its transpose matrix remain the same.

The idea of transpose of a matrix can be taught to the child through real life experience. For instance, in the classroom itself, a matrix with the students as the elements of some order can be formed, and then the rows and columns of students can be interchanged to form the transpose of the original matrix.

The idea can also be taught through the abacus. The usual abacus can form a matrix of order 4×15 (or 4×13 as the case may be) in the lower abacus horizontally, and the abacus when kept vertically will form a matrix of order 15×4, which is nothing but the transpose of the original matrix formed in the lower abacus.

Eg : Let, $A = \begin{pmatrix} 1 & -2 & -3 \\ 0 & 4 & -5 \end{pmatrix}$

Therefore, the transpose matrix of A denoted as A^T is given by,

$$A^T = \begin{pmatrix} 1 & 0 \\ -2 & 4 \\ -3 & -5 \end{pmatrix}$$

Note that in the above example, the order of matrix A is 2×3 and the order of its transpose matrix A^T is 3×2.

The idea may be taught with spatial presentation of the matrix supported by necessary text material in Braille.

16. Symmetric matrix

A matrix denoted as A is called a symmetric matrix if $A = A^T$.

Eg. : If $A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & -4 & 5 \\ -3 & 5 & 6 \end{pmatrix}$

Note that in the above mentioned matrix $A = A^T$ and hence A is a symmetric matrix.

The idea could be explained to the child with spatial presentation supported by necessary text material in Braille.
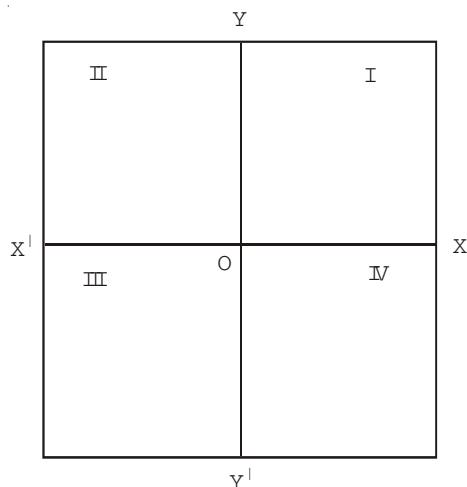
RECTANGULAR COORDINATE SYSTEM

1.   Rectangular coordinate system

The system of representation of points in the plane by ordered pair of numbers is called the rectangular or Cartesian or xy - coordinate system. The two axes are called rectangular or coordinate axes.

An embossed diagram depicting all the aspects of the rectangular system may be provided to the child to understand the idea.

2.   Quadrant

The term 'Quadrant' literally represents each of four parts of a circle, plane etc., divided by two lines or planes at right angles.

A plane can be divided into four quadrants by a pair of mutually perpendicular lines. The horizontal line is called the X- axis and the vertical line is called the Y- axis. The meeting point of the two axes is the origin denoted as 'O'.

A circular sheet of paper in which two straight lines cutting orthogonally at the centre enables the child to conceptualize that the two lines divide the circle into four equal parts and each is a Quadrant.

Provision of the concrete material on paper, can be supplemented by the verbal explanation of the teacher to enhance the learning of the child.

3. Origin

The point of intersection of the x-axis and the y- axis is the origin which is normally denoted by the letter 'O'. Note that the origin has coordinates (0,0).

Note : 1. Any point on the x-axis has its y-coordinate 0.

2. Any point on the y-axis has its x-coordinate 0.

3. All points on a line parallel to x-axis have the same y-coordinate.

4. All points on a line parallel to y-axis have the same x-coordinate.

The embossed diagram used for teaching the idea of rectangular coordinate system may be used for teaching the idea of origin also. Paper with dotted lines and geo-board may also be used for teaching these concepts.

4. Abscissa (X-coordinate)

Any point in the rectangular coordinate system is denoted as a set of points, a x-coordinate and the y-coordinate. The x-coordinate is also called as the abscissa.

The embossed diagram used for teaching the idea of rectangular coordinate system may be used for teaching the idea of origin also. A Braille sheet depicting a graph may be used to explain this concept.

5. Ordinate (Y-coordinate)

Any point in the rectangular coordinate system is denoted as a set of points, a x-coordinate and the y-coordinate. The y-coordinate is also called as the ordinate.

The embossed diagram used for teaching the idea of rectangular coordinate system may be used for teaching the idea of origin also.

6. Ordered pair

An ordered pair is a set of two elements mentioned in that order, the elements being written within parentheses. In analytical geometry, the x-coordinate and the y-coordinate together are enclosed within the parentheses and separated by a comma. Note that every point in the plane is represented as an ordered pair of real numbers. Also, the elements of an ordered pair are not interchangeable, that is ordered pairs (a, b) and (b, a) are not one and the same.

The idea of ordered pair may also be taught with the same embossed diagram depicting the rectangular coordinate system.

TRIGONOMETRY

Trigonometry

Trigonometry is a branch of mathematics which deals with the relations between the **measurements of sides and the angles of a triangle**. Hipparchus, a Greek astronomer and mathematician developed trigonometry and used its principles in predicting the paths and positions of the heavenly bodies. The word 'trigonometry' is derived from two Greek words **'trigon ' which means triangle and 'metra' which means measurement**.
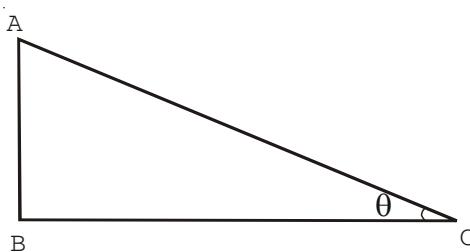
1. **Pythagoras theorem**
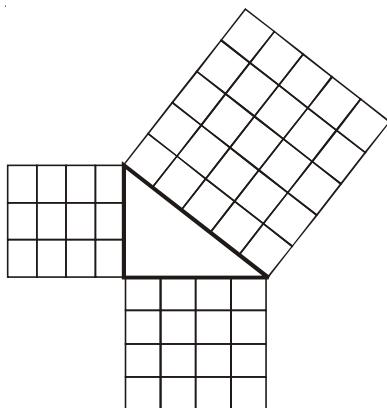
In a right angled triangle the area of the square constructed on the hypotenuse is equal to the sum of the areas of the squares constructed on the other two sides.

Let ABC be a right triangle in which the measure of angle ABC = 90^0. Then the side opposite to the right angle, AC is called the hypotenuse of the right triangle. Note that the hypotenuse is the longest side of a right triangle.

Therefore, according to the Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

A cutout or an embossed diagram may be used to explain this concept. Provide squares with dotted lines on the opposite and adjacent sides. Let the child count the total number of squares. Make a square with dotted lines for the hypotenuse too and let the child understand that the number of squares on the hypotenuse side are equal to the sum of squares on the opposite and adjacent sides.

2. **Trigonometric ratios**

Consider the right angle triangle ABC, where angle B = 90° and let us consider any one of the acute angles say angle A = theta. Then

$$\text{Sine} = \frac{\text{opposite side}}{\text{hypotenuse}} \left(\frac{AB}{AC} \right)$$
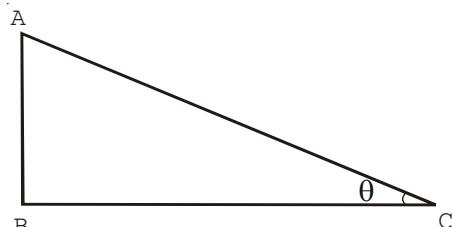
$$\text{Cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{Tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{Cosecant} = \frac{\text{hypotenuse}}{\text{opposite side}}$$

$$\text{Secant} = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$\text{Cotangent} = \frac{\text{adjacent side}}{\text{opposite side}}$$

The six trigonometric ratios namely sine, cosine, tangent, cosecant, secant and cotangent are normally written as sin, cos, tan, cosec, sec and cot respectively.

3. Trigonometric ratios of angles 0º, 30º, 45º, 60º,90º

0	0⁰	30⁰	45⁰	60⁰	90⁰
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

The trigonometric ratios of the specific angles mentioned can be taught through spatial presentation of this table in Braille format.

Trigonometric identities

1. $\sin\theta = \frac{1}{\cosec\theta}$

2. $\cos\theta = \frac{1}{\sec\theta}$

3. $\tan\theta = \frac{1}{\cot\theta}$

4. $\cosec\theta = \frac{1}{\sin\theta}$

5. $\sec\theta = \frac{1}{\cos\theta}$

6. $\cot\theta = \frac{1}{\tan\theta}$

7. $\sin\theta \cdot \cosec\theta = 1$

8. $\cos\theta \cdot \sec\theta$ = 1

9. $\tan\theta \cdot \cot\theta$ = 1

10. $\tan\theta$ = $\dfrac{\sin\theta}{\cos\theta}$

11. $\cot\theta$ = $\dfrac{\cos\theta}{\sin\theta}$

12. $\sin^2\theta + \cos^2\theta$ = 1

13. $1 + \tan^2\theta$ = $\sec^2\theta$

14. $1 + \cot^2\theta$ = $\csc^2\theta$

All the trigonometric ratios can be taught by keeping the embossed diagram and the table of values in front. Let the child understand specific values by referring to the diagram and the table back and forth.

Trigonometric ratios for complementary angles

15. $\sin(90° - \theta)$ = $\cos\theta$

16. $\cos(90° - \theta)$ = $\sin\theta$

17. $\tan(90° - \theta)$ = $\cot\theta$

18. $\cot(90° - \theta)$ = $\tan\theta$

19. $\sec(90° - \theta)$ = $\csc\theta$
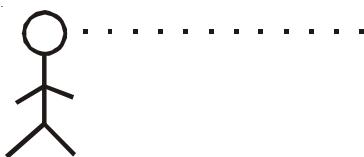
20. $\csc(90° - \theta)$ = $\sec\theta$

These ideas may be taught using logic. In the triangle, the three angles together will be equal to $180°$. Since a right angle triangle is used for defining sin, cos, tan, etc, in the

triangle ⟍ ABC, ∠B = 90⁰. Therefore ∠A + ∠C = 90⁰. Let ∠A = 30⁰, therefore sin 30⁰ = $\left(\dfrac{1}{2}\right)$. Since ∠A + ∠B = 90⁰, ∠B = 60⁰. Therefore sin (90⁰ − 30⁰) = sin 60⁰ which is $\left(\dfrac{BC}{AC}\right) = \dfrac{\text{opposite}}{\text{hypotenuse}}$, which is cos 30⁰ in which case BC becomes the adjacent side and AC is the hypotenuse. This logic may be applied to other trigonometic ratios too.
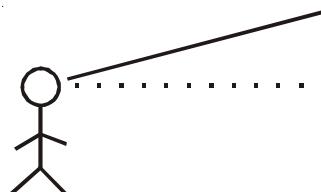
4. **Line of sight**

An imaginary line which is formed when a person is looking at an object without either looking up or down is said to be the line of sight or line of vision.

5. **Angle of elevation**
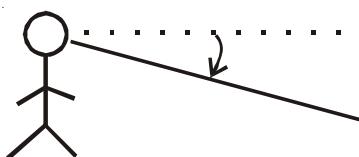
When an object above the line of sight is to be seen, the line of sight which is raised by an angle from the horizontal level is known as the angle of elevation.

6. **Angle of depression**

When an object below the line of sight is to be seen, the line of sight which is lowered by an angle from the horizontal level is known as the angle of depression.

All the three concepts - line of sight, angle of elevation and the angle of depression may be taught by an embossed diagram depicting the nature of the three.

ANALYTICAL GEOMETRY

Analytical Geometry

Analytical Geometry is a branch of mathematics in which the algebraic methods are used to study the geometrical concepts. De Cartes (1596-1650), a French mathematician invented this idea and hence analytical geometry is called as Cartesian Geometry, named after the inventor, who is considered as the father of analytical geometry.

1. **Angle of inclination**

    If a line makes an angle θ with the positive direction of the x-axis, then θ is called the angle of inclination.

    The idea needs verbal explanation along with the provision of an embossed diagram depicting the angle of inclination formed.
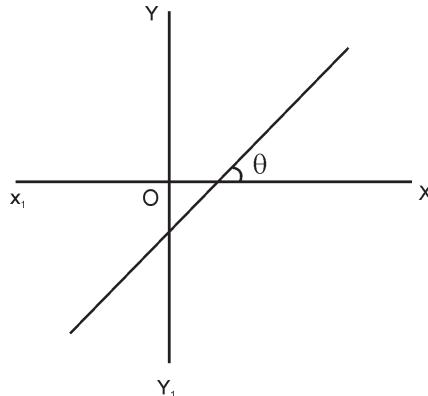
2. **Slope of a line**

    The tangent of the angle of inclination, that is tan θ is called the slope of the line or the gradient of the line. The slope of the line is denoted by m. That is slope = m=tan θ

    Note:

    1. *The angle θ is measured from the positive direction of the x-axis towards the line in the anti-clockwise direction.*
    2. *The slope of any line parallel to x-axis is zero since tan 0^0 = 0.*
    3. *The slope of any line perpendicular to x-axis is infinity since tan90^0 = undefined.*
    4. *If the angle θ is acute, then the slope tan θ is positive.*
    5. *If the angle θ is obtuse, then the slope tan θ is negative.*

The concept of slope of a line may be taught with an embossed diagram.
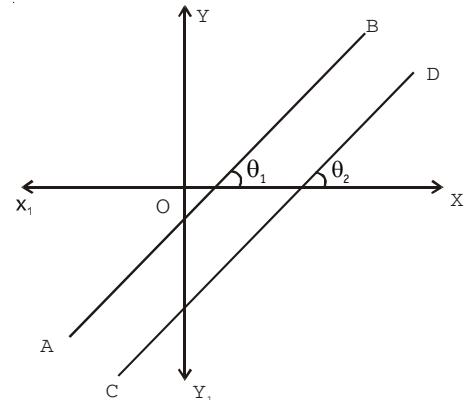
3. **Condition for two lines to be parallel**

Let AB and CD be any two parallel lines making angles θ_1 and θ_2 with the positive direction of the x-axis. Let their slopes be m_1 and m_2 respectively.

Therefore, $m_1 = \tan \theta_1$ and $m_2 = \tan \theta_2$. Since the lines AB and CD are parallel to each other, $\theta_1 = \theta_2$ (corresponding angles)

Therefore, $\tan \theta_1 = \tan \theta_2$

That is $m_1 = m_2$

Hence if two lines are parallel they have the same slope.

As the idea of "parallel lines" has already been taught the child needs only some additional input regarding parallel lines in terms of slope of a line.

4. **Condition for two lines to be perpendicular**

If two lines are perpendicular to each other then the product of their slopes equals -1. That is, if two lines are perpendicular to each other their slopes are negative reciprocals of each other; conversely, if the slopes of any two lines are negative reciprocals of each other, the lines are perpendicular.

As the child is already familiar with the idea of perpendicularity, its nature relating to slope of a line alone needs explanation.

5. Equation of a straight line

The equation of any line passing through the origin is of the form $y = mx$. If a straight line intersects the axes then the line segments cut off on the axes by the line are called the intercepts.

There are several forms of equations of straight lines of which the following are few.

(i) Slope intercept form

The equation of a line with slope m and y – intercept c is given by

$$y = mx + c$$

(ii) Slope - one point form

The equation of a line passing through a point (x_1, y_1) and having slope 'm' is,

$$y - y_1 = m(x - x_1)$$

(iii) Two points form

The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2) is given by,

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
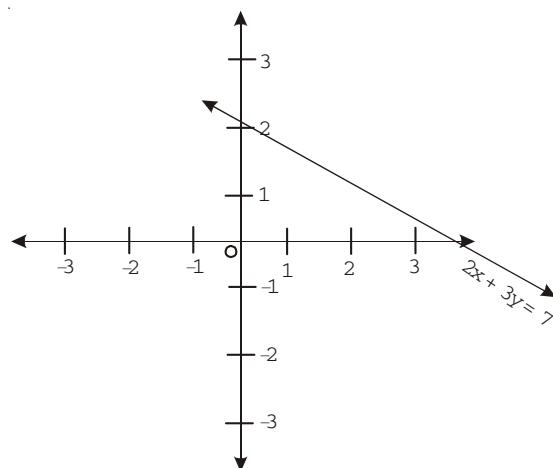
(iv) Intercepts form

The equation of the line having x-intercept 'a' and y- intercept 'b' is given by,

Note :

The four equations mentioned above are all first degree equations in x and y. Note that the general first degree equation in x and y always represents a straight line. Hence the general equation of a straight line is ax + by + c = 0, with atleast one of a or b is different from c.

Eg. : Graph of 2x + 3y = 7
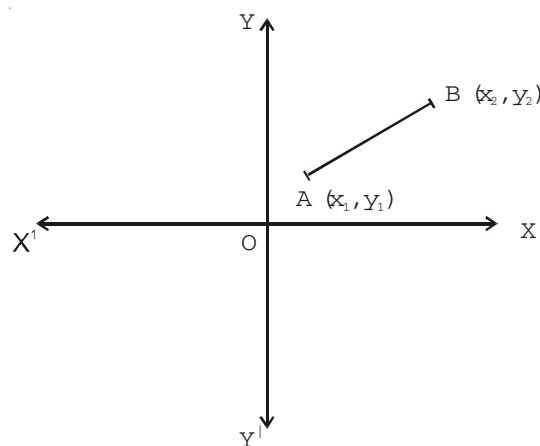
6. Distance formula

In the rectangular coordinate system, the distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by,
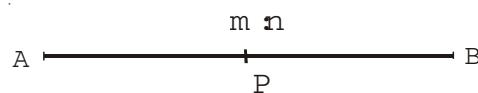
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The idea can be taught with linear presentation of the equation alongwith the embossed rectangular coordinate system for additional input.

7. Section formula

Consider the line AB whose coordinates are $A(x_1, y_1)$ and $B(x_1, y_1)$. Let "P" be a point which divides the given line in the ratio m:n. Then the coordinates of the point which divides the line segment AB can be found.

Internal division:

The coordinates of the point P is $\left(\dfrac{mx_2 + nx_1}{m+n}\right), \left(\dfrac{my_2 + ny_1}{m+n}\right)$

External division:

The coordinates of the point P is $\left(\dfrac{mx_2 + nx_1}{m-n}\right), \left(\dfrac{my_2 + ny_1}{m-n}\right)$

8. Mid point formula
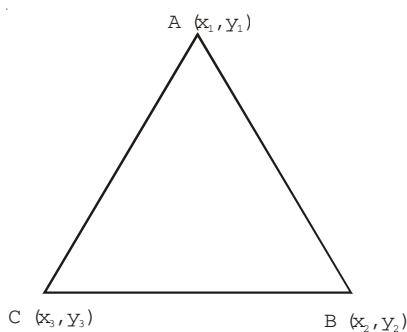
Let A (x_1, y_1) and B(x_2, y_2) be any two points. Then the midpoint of the line AB is given by,

Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

The idea may be explained with the provision of an embossed number line and Braille text material.

9. Area of a triangle

Let A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) be the vertices of a triangle. Then the area of the triangle ABC is given by,
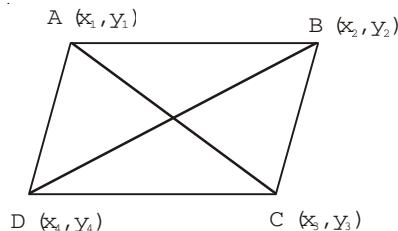
Area of the triangle ABC $= \dfrac{1}{2} \{ x_1(y_2\text{-}y_3) + x_2(y_3\text{-}y_1) + x_3(y_1\text{-}y_2)\}$

The idea may be explained with the provision of Braille text material.

10. Area of a quadrilateral

Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$, $D(x_4,y_4)$ be the vertices of the quadrilateral ABCD. Then the area of the quadrilateral will be equal to the two triangles which are obtained upon drawing a diagonal AC for the quadrilateral. Then,

Area of the quadrilateral ABCD = Area of triangle ADC + Area of triangle ACB

The formula used for triangle be applied to find out the area.

11. Condition for collinearity

Three or more points are said to be collinear if they lie on the same straight line. The condition for the three points (x_1,y_1), (x_2,y_2), (x_3,y_3) to be collinear is the area of the triangle formed by the three points should be equal to zero.

That is, $x_1(y_2-y_3) + x_2(y_3-y_2) + x_3(y_1-y_2) = 0$

Alternate method:

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be the three given points. To check whether the given points are collinear, it is enough to show that,

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$$
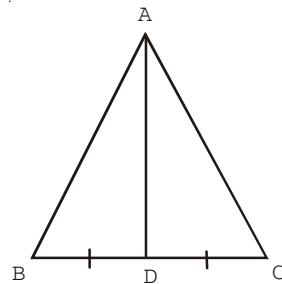
At this stage, the child is familiar with the idea of collinear already, and before, the above example may be easy to understand.

12. Median

Median is the line passing through the midpoint of a side to the opposite vertex. Note that three medians can be drawn to a triangle and all the three medians pass through the same point.
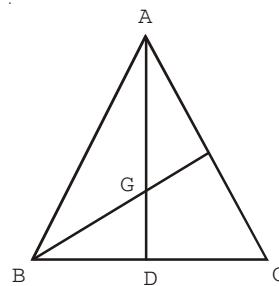
The idea of median may be explained to the child using paper folding. As the child himself is facilitated to perform the activity in finding the medians of a triangle, he/she may understand the idea effectively.

13. Centroid

The point of concurrence of the medians of a triangle is called the centroid of the triangle. Note that the centroid of a triangle divides the medians in the ratio 2:1. Centroid of a triangle is normally denoted by the letter G.

The concept of centroid may be taught to the child through the paper folding. The procedure to be followed is mentioned in the creative mathematics section.

14. Altitude
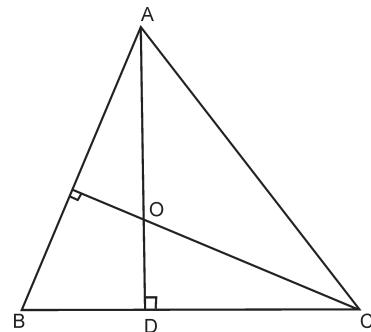
An Altitude of a triangle is a perpendicular line segment from a vertex of the triangle to the opposite side.

The idea may be taught with the assistance of paper folding.

15. Orthocenter

The point of concurrence of the altitudes of a triangle is called the orthocenter and is normally denoted by the letter 'O'.
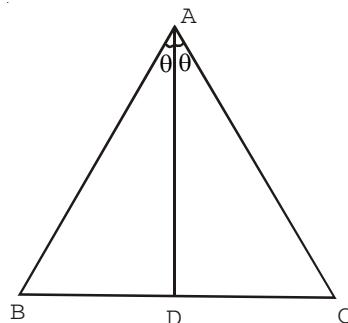
The idea may be taught orally with the assistance of paper folding.

16. Angle bisector

A line which divides the angle of a triangle into two halves is called the angle bisector.
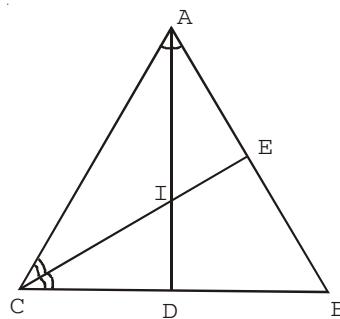
The idea of angle bisector may be taught with the assistance of paper folding.

17. Incenter

The point of concurrence of the angle bisectors of a triangle is called as the incenter of a triangle and is normally denoted by the letter I.
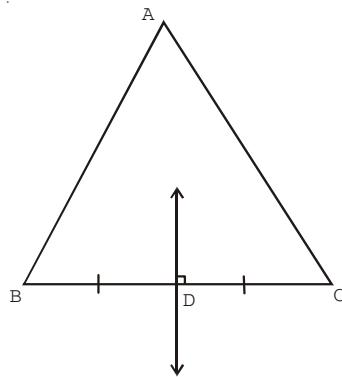
Once the child is clear with the angle bisectors, the idea of incenter may be taught orally with the assistance of necessary Braille material assisted by paper folding.

18. Perpendicular bisector of the side of a triangle

A line (ray or segment) which is perpendicular to a segment at its midpoint is called the perpendicular bisector of that segment.
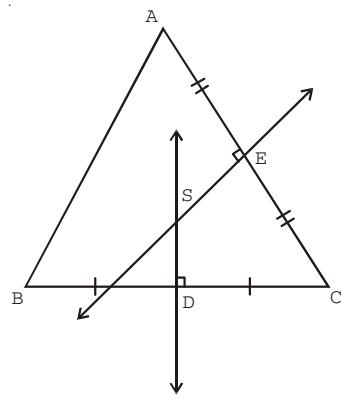
Note that the ray may not pass through vertex A. Paper folding techniques will be useful to teach this concept.

19. Circum center

The point of concurrence of the perpendicular bisectors of the sides of a triangle is called as the circum center of the triangle and is normally denoted by the letter S.

Once the child is clear with the nature of perpendicular bisector, the idea of circumcenter may be taught with the assistance of paper folding.

Module 83

STATISTICS

Statistics

Statistics is a branch of mathematics which deals with the collection, presentation, analysis and interpretation of the numerical data.

The word 'Statistics' has been derived from the Latin word 'Status' which means a political state.

The word 'statistics' is used both in singular and plural sense. In the singular sense, statistics is a science which deals with the collection, presentation, analysis and interpretation of the numerical data. In the plural sense, statistics means all the numerical data collected with some purpose.

1. **Data**

   Data are a number of facts. The word 'datum' is singular and 'data' is plural. Datum is a Latin word which means 'fact'. The process of gathering the data for some specified purpose is called data collection.

   The idea may be taught orally.

   Eg. : The marks scored by 10 students in an examination are :

   45, 90, 86, 74, 57, 36, 98, 27, 64, 50

2.   Population

The set of all observations or objects in a particular category or group is called population.

Eg. : People, Animals, Countries, etc.
The idea may be taught orally with the provision of text material in braille.

3.   Sample

The sample is the true representative of the population.

Eg. : If students of a particular grade, say X, studying in a region is the population, then the students of grade X studying in a particular school of the region may be treated as a sample.

4.   Primary data

When data are collected from the elements of the population, that is when the data is collected from the primary source, it is called the primary data.

Eg. : In a particular class of students, if the heights of the students are measured and noted then it is a primary data.

5.   Secondary data

The data obtained from a secondary source, not collected directly from the population, are called secondary data.

Eg. : In the same class of the students, if the heights of the students are noted based on the earlier records maintained by the school, then such a collection is said to be a secondary data.

6.   Raw data

Data presented in the form as it is collected, is called raw data or ungrouped data.

Eg. : The marks obtained by 20 students of grade 10 of a particular school in a maths test is as follows:

47, 58, 94, 87, 60, 75, 34, 27, 82, 63, 56, 50, 59, 40, 28, 12, 18, 77, 69, 10.

In the above data, no classification is made and hence it is a raw data.

7. Grouped data

Data represented in various groups or classes is called grouped data.

Eg. : Consider the data in item 6. The data can be classified as follows:

C-I	Frequency
1-20	3
21-40	4
41-60	6
61-80	4
81-100	3

8. Class interval

After the collection of the numerical data they are classified into suitable groups as per the convenience. The groupings with a lower limit and a upper limit are called as the class intervals. The size of the class intervals should be equal in size. The size of a class interval is also called as class width. The middle value of the class interval is called the class mark.

Eg. : In the above example, all class intervals 1-20, 21-40, 41-60, etc., are of equal size.

9. Frequency

The number of observations which fall in one particular class interval is called the frequency of the class interval. To determine the frequency of each class interval tally marks are made for each datum in that class till all the data are exhausted.

Eg. : In the example in item 7, the frequency of the class interval 41-60 is 6.

10. Frequency distribution table

Grouped data presented in a table form listing class intervals and corresponding frequencies is called a frequency table.

Eg. : *Consider the data in item 6. The data can be grouped in the form of a frequency distribution table as follows:*

C-I	Tally Marks	Frequency
1-20	|||	3
21-40	||||	4
41-60	ⅢⅠ	6
61-80	||||	4
81-100	|||	3

Present this table in the tactile form. Counting beads, punching braille dots on paper, etc., may also be used as techniques to tabulate data in frequency table.

PICTORIAL REPRESENTATION

11. Bar chart

The bar chart consists of bars of equal thickness with the length/height being proportional to the quantity the bars represent. Bar charts are of various types such as vertical bar chart, horizontal bar chart, double bar chart, etc.
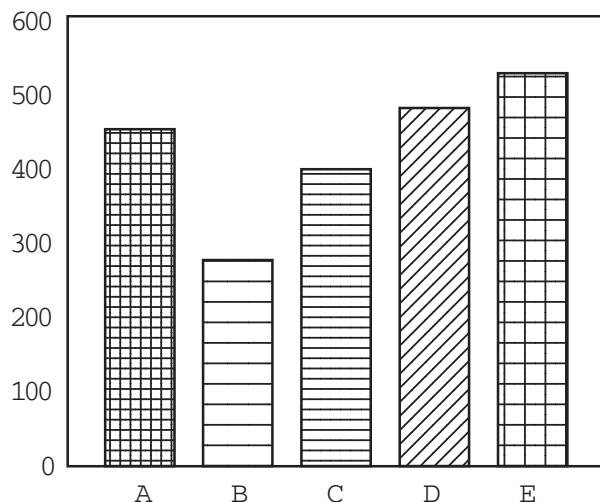
Note :
1. *The bar-chart must have a heading.*
2. *Sub-divisions are to be made on the vertical axis, using a suitable scale.*
3. *Bars are of equal thickness and do not touch each other.*
4. *Each bar is assigned a heading to denote what it represents.*
5. *The height of each bar represents the number it is supposed to represent.*

An embossed bar chart prepared on a thick sheet of paper may be provided to the child for exploration and the features of the bar chart are to be explained simultaneously.

Eg. : *The number of students studying in 5 schools in a city are as follows:*
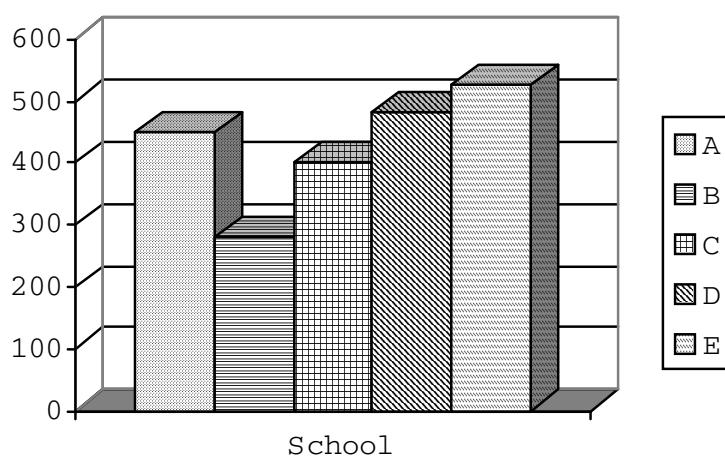
School	A	B	C	D	E
No. of students	450	280	400	480	525

No. of students studying in different schools

12. Histogram

The histogram is closest to the bar graph. If the bars in a graph drawn are touching each other, then a histogram is obtained. The histogram is composed of a set rectangles one for each class interval on the horizontal scale. The histogram is drawn when a frequency distribution is given. The steps of drawing a histogram are the same as that of the bar chart. The idea needs to be taught with the provision of an embossed chart depicting the construction of an histogram for some data.

Eg. : The Histogram for the data mentioned in item No.11 is as follows:
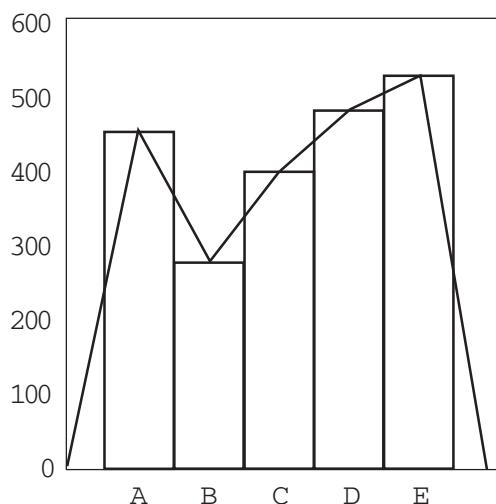
No. of students studying in different schools

13. Frequency Polygon

A frequency polygon is obtained by joining the mid points of the upper sides of the bars of a histogram by means of line segments.

If the child is through with the idea of histogram he/she may be taught of the nature and construction of a frequency polygon. The relation between a histogram and a frequency polygon need to be emphasized in terms of their similarities and differences.

Eg. : *The Frequency Polygon for the data mentioned in item No.11 is as follows:*

14. Pictogram

Pictogram is a way of representing an information through pictures or symbols. When a numerical data is represented in the form of pictures or symbols the idea may be easily understood.

A pictogram depicting some numerical data may be prepared and be given to the child for exploration.

Eg. : *The Pictogram for the data mentioned in item No.11 is as follows:*
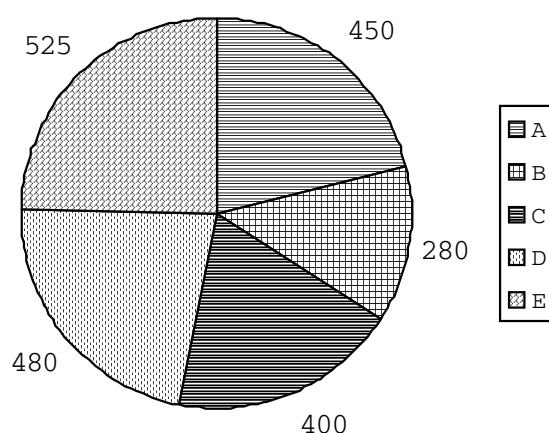
A (450)

B (280)

C (400)

D (480)

E (525)

15. Pie Chart

Another mode of representing the data is through the use of a pie chart in which the entire circle is taken to represent a whole and all the constituents of the entire data are represented proportionally in the pie chart.

Conversion of the given data to degree measures is to be explained to the child initially. Once the child is clear with the conversion he/she needs to be explained about the construction of a pie chart. Construction of a pie chart by the child, may not be possible but the child can understand all the relevant ideas relating to a pie chart.

Eg. : *The Frequency Polygon for the data mentioned in item No.11 is as follows:*
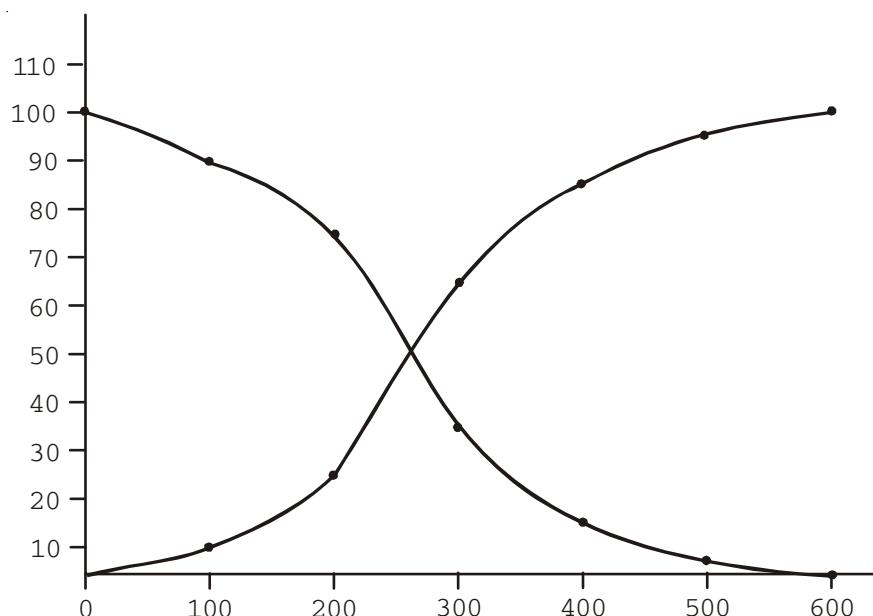
16. Ogive

The cumulative frequency of a particular class is obtained by adding all the frequencies up to and inclusive of that class interval. An ogive is a graph of cumulative frequency distribution. It is also called frequency distribution curve. To draw a cumulative frequency distribution curve, plot the points with the upper limits of the classes on the x-axis and the corresponding cumulative frequencies on the y-axis.

The nature and construction procedures regarding an ogive, need a thorough verbal explanation in addition to the provision of an embossed ogive prepared for some data.

Weekly wage in Rs.	No. of Workers	Cumulative Frequency for less than	Cumulative Frequency for more than
0 - 100	10	10	100
100 - 200	15	25	90
200-300	40	65	75
300 - 400	20	85	35
400 - 500	10	95	15
500 - 600	5	100	5

17. Measures of Central Tendency

In many large groups of data, the observations tend to cluster around some central value which is called as the central tendency. The three basic measures of the central tendency are the mean, median and mode.

18. Mean (For a raw data)

Mean is the average value of a set of data. In other words, mean is the sum of all the data divided by the total number of observations and is denoted as X.

$$\text{Mean} = \frac{\text{Sum of all the data}}{\text{Total number of observations}}$$

Eg. : Find the mean of 4, 7, 8, 1, 5

Mean = Sum of all quantities/Total number of quantities

$$= \frac{4 + 7 + 8 + 1 + 5}{5}$$

$$= 5$$

The concept can be taught easily if the child has mastery over addition and division.

19. Assumed mean

To avoid multiplication by big numbers, a number is assumed to be the mean and the difference of individual data from the assumed mean is found. The arithmetic mean is the sum of the assumed mean and the average of the differences.

Arithmetic mean = Assumed mean + $\frac{\sum d}{n}$ (ungrouped data) or, Arithmetic mean = Assumed mean + $\frac{\sum fd}{n}$ (ungrouped data) where 'd' is the deviation of the items from assumed mean.

Eg. : Compute the A.M. of the following data:

x	10	11	13	15	16	19
f	4	5	8	6	4	3

Take Assumed mean, A = 14, Class interval, c = 1, Deviation, d = x –A

x	f	d	fd
10	4	–4	–16
11	5	–3	–15
13	8	–1	–8
15	6	1	6
16	4	2	8
19	3	5	15
Total	**N=30**		$\sum$ **fd = –10**

$$\bar{x} = A + c \times \frac{\sum fd}{N} = 14 + 1 \times \frac{-10}{30}$$

$$= 14 - 0.33 = 13.67$$

20. Median (for a raw data)

Median is the value which divides the given data exactly into two halves i.e., median is the middle most value wherein one half of the data lies above it and the other half lies below it, when arranged either in ascending or descending order. If 'n' is odd then the median is $\frac{n+1}{2}$ th value and if 'n' is even the median is the mean of $\frac{n}{2}$ and $\frac{(n+1)}{2}$ th values.

Provide the child with a set of wooden strips, odd in number. Ask the child to arrange them, either in ascending or descending order on the basis of their length/height. Now ask the child to locate the middle strip which divides the given set of strips exactly into two halves. The middlemost strip thus identified becomes the median in the set of strips.

Note that in the case of total number of data being even, the average of two values in the middle becomes the median.

Eg. :   Find the median : 12, 54, 13, 47, 38

In ascending order : 12, 13, 38, 47, 54

Here n = 5, odd.

Therefore, median $= \left(\dfrac{n+1}{2}\right)^{th}$ item $= \left(\dfrac{5+1}{2}\right)^{th}$ item $= \left(\dfrac{6}{2}\right)^{th}$ item $= 3^{rd}$ item $= 38$

21. Mode

Mode is the value which occurs most frequently in a data. To find the mode in a given data, it is just enough to identify the value which occurs the most number of times. If no number occurs more than once then there is no mode. If more than one value occurs the same number of times in a data, then all such numbers become the modes.

To enable the child with visual impairment to understand the concept, explain the idea verbally and then provide the child with a set of articles which contain a particular object more than once, say thrice. Then, that particular object becomes the mode among the set of articles chosen.

Eg. : Find the mode : 2, 5, 7, 8, 2, 3, 4, 6, 2, 10
      Here 2 occurs thrice and hence 2 is the mode.

22. Measures of Dispersion

The measures of dispersion help in finding out the extent of dispersion of the observations in the data. The commonly used measures of dispersion are the range, standard deviation and the variance.

23. Range

The range is the difference between the maximum value and the minimum value in a given data.

Therefore, Range = Maximum value - Minimum value

Range is also called the spread.

Eg. : Find out the range of the data : 27, 28, 34, 36, 39, 59

Range = Maximum value – Minimum value

= 59 – 27 = 32

24. Standard deviation

Standard deviation is the square root of the mean of the squares of the differences of individual scores from the mean. Standard deviation is denoted by the Greek letter σ (Sigma). It can be obtained by the formula

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

It is also obtained using the formula, $\sigma = \sqrt{\dfrac{\Sigma d^2}{n}}$

where $d = x - \bar{x}$ and $\bar{x}$ is the mean.

The idea of standard deviation may be explained orally laying emphasis on the memorization of the formula and knowing of its components. A thorough explanation of the idea with the aid of a model problem will be useful to explain this concept.

x	x^2
14	196
22	484
9	81
15	225
20	400
17	289
12	144
11	121
x = 120	$\Sigma\ x^2 = 1940$

$$\sigma = \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$$

$$= \sqrt{\dfrac{1940}{8} - \left(\dfrac{120}{8}\right)^2}$$

$$= \sqrt{242.5 - 225}$$

$$= \sqrt{17.5}$$

$$\sigma = 4.18$$

25. Variance

Variance is the square of the standard deviation.

Variance, $\sigma = \left(\sqrt{\dfrac{\sum d^2}{n}}\right)^2 = \dfrac{\sum d^2}{n}$

Once the child is through with the idea of standard deviation, the concept of variance can be introduced stating clearly the relation between the two.

Eg. : Variance = (Standard deviation)^2

Hence in the previous example, Variance = 17.5

PROBABILITY

1. **Experiment**

   An experiment is an activity carried out in anticipation of a result out of it. Tossing a coin, rolling a die and picking a card from a pack of cards, etc., are examples of an experiment.

   Real life experience can be provided by asking the child to either toss a coin or roll a die and explaining all the related ideas simultaneously.

2. **Random experiment**

   An experiment is called a random experiment when conducted repeatedly under essentially homogenous conditions. In other words, an experiment whose outcome cannot be predicted is called as a random experiment.

   The idea may be explained orally and the child may be helped to perform a activity of his/her own.

3. **Trial**

   Performing a random experiment is called a trial.

   The concept may be taught by helping the child to perform the activity of his/her own - tossing the coin, selecting an object from a pile, etc.

4. **Event**

   The outcome of a trial in a random experiment is termed as an event. Once the child is clear with the idea of a trial, he/she may explained the idea of an event.

5.   Outcome

The outcomes of the experiment are also known as sample points.

As the child is aware of the ideas of trial and event the idea of outcome may be understood easily.

6.   Sample space

The set of all possible outcomes of a random experiment is known as the sample space and is denoted by S.

Eg. : In rolling of a die, the possible outcomes are 1, 2, 3, 4, 5 and 6.

Therefore, sample space S = {1, 2, 3, 4, 5, 6}

The child must be clearly explained that the totality of all the outcomes is the sample space.

7.   Equally likely events

Two or more events are said to be equally likely if each one of them has an equal chance of happening.

Eg. : In tossing of an unbiased coin, getting head or tail are equally likely events.

8.   Exhaustive events

All possible outcomes of a random experiment are called as exhaustive events, also known as sample space.

Eg. : In tossing of a coin the possible outcomes are head and tail.

$S = \{H, T\}$

Number of exhaustive events = 2

9.   Mutually exclusive events

Two or more events are said to be mutually exclusive if the happening of one of them prevents the occurrence of all the other events in the same experiment.

Eg. :   *In tossing a coin getting a head or getting a tail are mutually exclusive events, because the occurance of head automatically prevents the occurance of tail.*

10. Mutually independent events

Two or more events are said to be mutually independent if the occurrence of one event does not prevent the occurrence of the other events in the same experiment.

Eg. :   *In tossing two coins, getting a head or tail in the first coin, and getting a head or tail in the second coin are mutually independent events, because the occurance of one event cannot prevent the occurance of the other.*

The child must be facilitated to make a clear distinction between mutually exclusive and mutually independent events.

11. Probability

In a random experiment, the probability of occurrence of the event A, denoted by P(A) is defined as,

$$P\ (A)\ =\ \frac{\text{Number of favourable outcomes for A}}{\text{Number of exhaustive outcomes}} = \frac{n(A)}{n(S)} = \frac{m}{n}$$

Note:

1.   *The probability of an event A lies between 0 and 1 i.e., $0 \leq P(A) \leq 1$.*

2.   *Number of outcomes which are not favorable to the event A = n-m. Probability of the non-occurrence of A denoted by A' is given by*

$$P(A')\ =\ \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$P(A')\ = 1 -\ P(A)$$

Therefore, P(A) + P(A') = 1

That is, the sum of the probability of the occurrence of an event and its non-occurrence is equal to 1.

The concept of probability, needs a thorough verbal explanation in addition to the use of live experiments like tossing coin, selecting objects from a pile, using lot system for sample selection, etc.

12. Sure(certain) event

An event is said to be a sure event (or a certain event) if the probability of the occurrence of the event is equal to 1.

Eg. : In the event of tossing of a coin, the occurance of a head or tail is a sure event.

13. Impossible event

An event is said to be an impossible event if the probability of the occurrence of the event is equal to 0.

Eg. : Consider the probability of getting the face number 7 in rolling of a die. The probability of the occurrence of this event is equal to zero and hence it is an impossible event.

14. Addition theorem on probability

If A and B are two mutually independent events then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are two mutually exclusive events then, $P(A \cap B) = 0$, and hence the addition theorem on probability becomes,

$$P(A \cup B) = P(A) + P(B)$$

The theorem may be explained supported by relevant text material in Braille. The examples used in the section dealing with set theory may be used to describe this concept.

15. Multiplication theorem on probability

If A and B are any two independent events then the probability of simultaneous happening of two independent events is equal to the product of their individual probabilities.

That is, $P(A.B) = P(A). P(B)$

Going through the concepts described in this section must have provided you adequate knowledge in mathematics usually prescribed for secondary level education and also with the techniques of adapting for children with visual impairment.

Section 5

Creative Mathematics

CREATIVE MATHEMATICS

As children with visual impairment are devoid of one of the vital senses, the vision, the vacuum created has to be compensated by some other means to enable them to compete with their sighted peers in the academic activities. The tactile sense lends a helping hand for a child with visual impairment to complement the loss of vision at least to some extent. In teaching a concept, any object or thing which is tactile in nature always attracts the child with visual impairment.

Mathematical concepts, though abstract in nature when presented in concrete form arouse and sustain interest among persons, both sighted and visual impairment. Creative mathematics through paper folding enables children with visual impairment to understand the abstract concepts with ease. The paper folding leaves a crease which is very much tactile in nature and enables a child with visual impairment to learn the concept at a faster pace. The concept of creative mathematics through paper folding is in line with the basic objective in development of a teaching aid that it is cheaper, it can be used well and be changed often also. The amount of impact that a paper folding creates in knowing of mathematical concepts is so vital that it can be used for teaching a score of concepts, which are otherwise felt as very much abstract in nature. For instance, an abstract concept in Geometry, say "the angle formed in a semicircle is always a right angle" can be taught to the child with visual impairment through paper folding in the following ways.

Step 1 : Take a thick sheet of paper and cut it in the form of circle, to any convenient radius, say 10 cm.

Step 2 : Locate its centre and fold the circle so as to form a diameter.
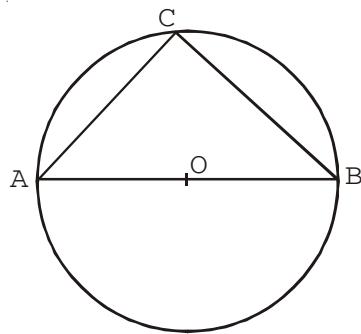
Step 3 :   The crease thus formed divides the circle into two halves thus making two semicircles.

Step 4 :   Select any one semicircle and fold the paper twice appropriately so as to form an angle at the circumference of the circle.

Step 5 :   Place any one of the corners of a neatly cut paper on the angle formed at the circumference, and also help the child to tactually explore that the angle so formed is a right angle.

Step 6 :   Similarly, create any number of angles in the semicircle and observe that each of the angles is equal to a right angle.

The above mentioned example reveals that how an abstract concept can be presented to the child with visual impairment in concrete form. Similarly, a number of abstract concepts can be taught by creative mathematics in the form of paper folding.

Module 86

ANGLES

1. Line Segment

A ├─────────────────────┤ B
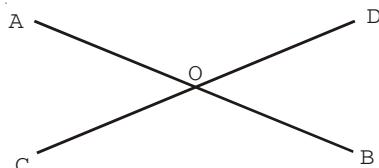
Take a paper and fold it across. The impression created is a line segment having only length and no width or height.

2. If two line segments intersect they will intersect at only one point

Take a paper and fold it across. Fold once again so that two intersecting lines are formed. The two line segments thus formed can intersect at one single point only.
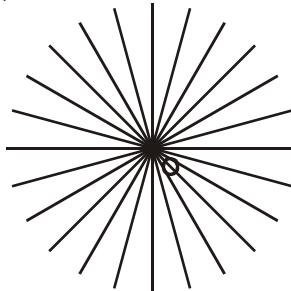
3. Through two points only one line segment can be drawn

Plot two points on a sheet of paper. Fold the paper as such the impression created is along the two points. The impression created reveals that through two points only one straight line segment can be drawn.
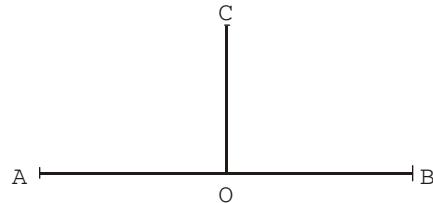
4. Through a point infinite number of lines can be drawn
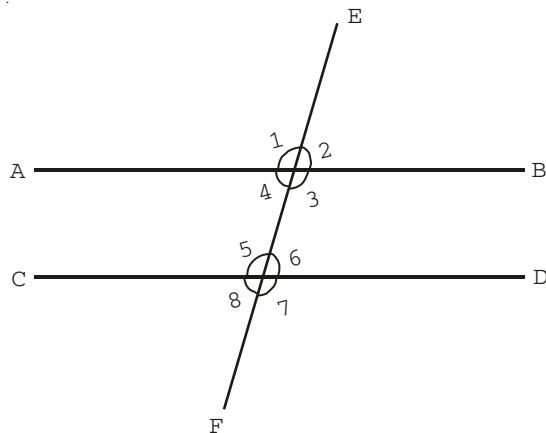
Plot a point approximately at the centre on a sheet of paper and fold the paper in as many ways as possible so that all the lines pass through that point. Any number of lines can be formed and this implies that through a point infinite number of lines can be drawn.

5. Perpendicularity

Take a neatly cut paper which is in the form of a square or rectangle. Fold the paper vertically so as to form two halves. Now fold it once again horizontally to form halves on the other way. The impressions thus formed are perpendicular to each other.

6. Parallel lines & Transversal
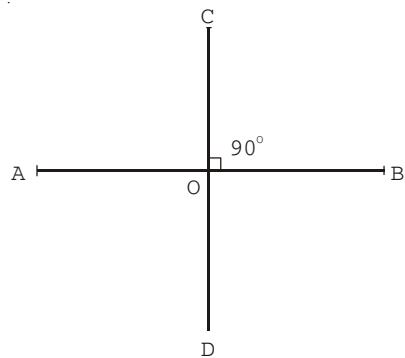
Fold the paper, so that it forms a straight line. Fold once again, in such a way that the impression created is equidistant from the previous impression at all the points. The impressions created form two parallel lines.

If a third line is formed in such a way that it cuts the earlier two lines at two different points then this line becomes a transversal.
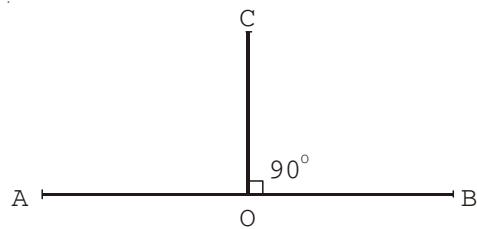
Note : A line which cuts two other lines at two different points is called a transversal.

7. Perpendicular bisector
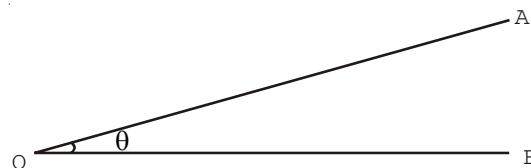
Fold the paper vertically to form exactly two halves. Similarly, fold again horizontally. The two impressions created are perpendicular bisectors to each other.

8. Right Angle
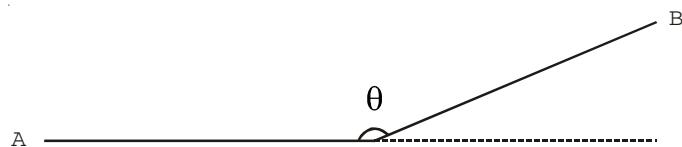
Any neatly cut paper will form a right angle at its vertices.

9. Acute Angle
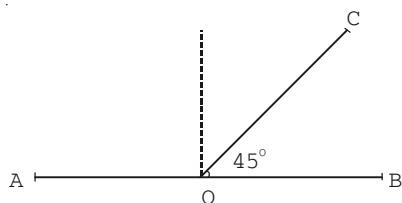
Cut a paper in such a way that one of the angles formed is less than a right angle. The angle so formed is an acute angle.

10. Obtuse Angle
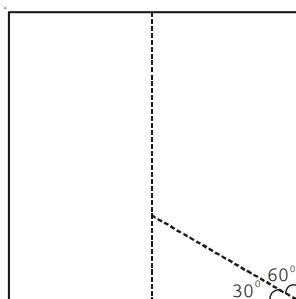
Cut a sheet of paper in the form of a triangle in such a way that one of the angles formed is more than a right angle. The angle so formed is an obtuse angle.

11. Angle 45^0
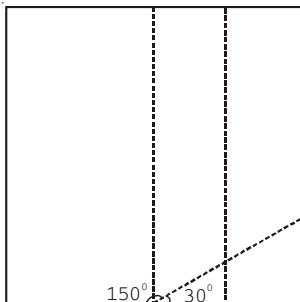
Any paper which forms 90^0 at its vertices, when folded along its vertex once again forms 45^0 on either side of the impression

12. Angle 60^0, 30^0

Fold a sheet of paper exactly into two halves, either lengthwise or widthwise. Now bring one lower corner of the paper to touch the crease formed at the middle and fold it accordingly to make another crease dividing the lower corner into two parts. The two angles formed on either side of the crease thus making angles of 30^0 and 60^0 respectively.

13. Angle 150⁰
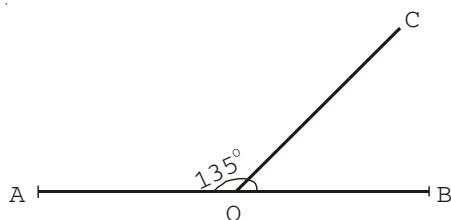
Fold a neatly cut paper into two halves either lengthwise or widthwise. Now fold the one-half again to make two equal parts, each of which is one-fourth of the original paper. Select the portion in which the one-half is divided into two. In that portion, follow the procedures of obtaining angles measuring 30⁰ and 60⁰. Thus the folding has two angles 30⁰ on one side and 150⁰ on the other.

14. Angle $22\frac{1}{2}^0$

Take a neatly cut paper which forms 90⁰ on all corners. When the vertex is halved it forms 45⁰ on either side. If the 45⁰ measure is halved again it forms $22\frac{1}{2}^0$.

15. Angle 135⁰
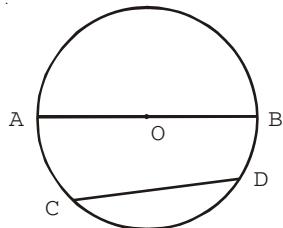
Take a sheet of paper and fold it across, forming 90⁰ on either side of the impression. Choose one half of the paper and fold the 90⁰ measure again into two halves to form 45⁰ each on either side. Thus the two impressions created form 135⁰ on one side and 45⁰ on the other.

CIRCLES
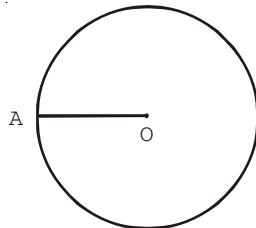
1. Chord, diameter
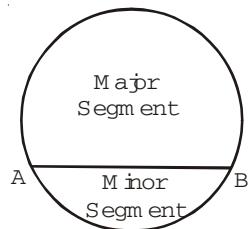
Cut a sheet of paper in the form of a circle. Fold the paper to divide the circle into two parts in many ways. Each impression created is a chord. If the folding is through the centre then the impression becomes a diameter.

2. Radius
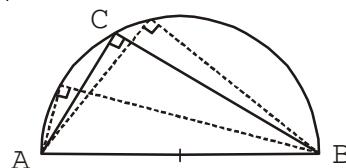
Cut a sheet of paper in the form of a circle. Fold the circle to form impressions from the centre to the circumference of the circle. Each one is a radius of the circle.

3. Circle – Minor Segment and Major Segment
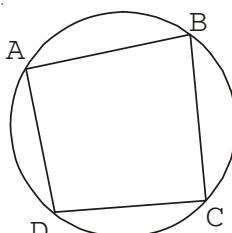
Cut a sheet of paper in the form of a circle. Fold the circle so that the impression created forms two parts one greater than the other. Here the greater part is a major segment and the smaller is a minor segment.

4. Angle in a Semi Circle is 90°
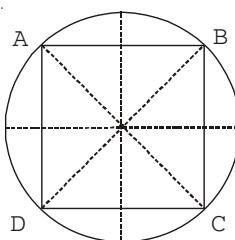
Cut a sheet of paper in the form of a circle and then fold it along any diameter to form two semicircles. Fold the paper twice appropriately from both the ends of the diameter to meet at the circumference of the circle. Observe that the angle so formed in the semicircle is always a right angle. Also note that all angles formed at the point of the circumference are 90^0.

5. Cyclic Quadrilateral

Cut a sheet of paper in the form of a circle. Fold the circle to form four chords so that one chord touches the other two at their extremities thus forming a cyclic quadrilateral.

6. Square in a Circle
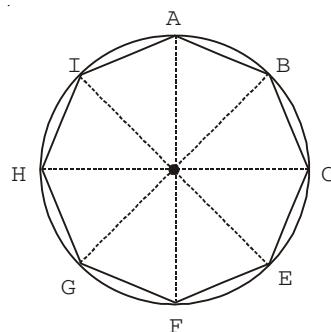
Cut a sheet of paper in the form of a circle. Fold the paper twice in such way that two diameters cutting orthogonally are formed. Now fold the paper four times forming
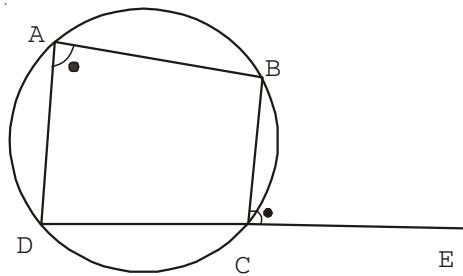
lines across the extremities of the two diameters. Observe that a square is formed inside the circle.

7.   Regular Octagon in a Circle
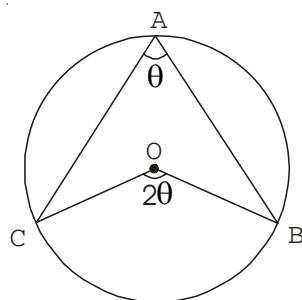
Cut a sheet of paper in the form of a circle. Fold the paper twice so that two diameters cutting at right angles are formed. Fold the paper again twice so that the four right angles formed are further halved into eight angles of 45° each. Now fold the paper eight times so that all the extremities of the diameters are connected. Observe that a regular octagon is formed inside the circle.

8.   In a cyclic Quadrilateral the exterior angle is equal to the interior opposite angle

Construct a cyclic quadrilateral in a sheet of paper. Extend any one of the sides of the quadrilateral beyond the circumference of the circle. Observe that an angle, called exterior angle is formed between the side extended and one of the chords. Now cut off the exterior angle from the cyclic quadrilateral. Place the part cut off coinciding with the interior opposite angle and observe that the exterior angle is equal to the interior opposite angle. That is angle BCE is equal to angle DAB.

9. **In a circle the angle subtended at the centre is twice that of the angle subtended at the circumference**
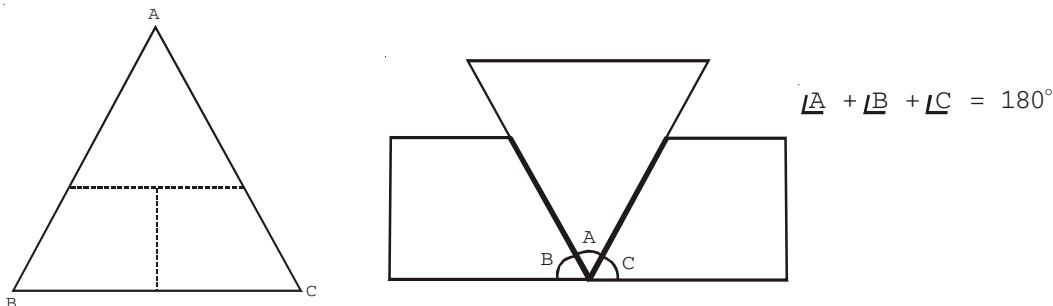
Cut a sheet of paper in the form of a circle. Fold the paper two times to form two radii subtending some angle at the centre. Now fold the paper again twice from the extremities of the two radii to form another angle at the circumference of the circle. Create a similar circle and cut off the sector alone. By folding the sector along the radii from the midpoint of the arc and placing the folded paper on the angle formed with circumference in the paper intact, observe that in a circle the angle subtended at the centre is twice that of the angle subtended in the circumference.

TRIANGLE
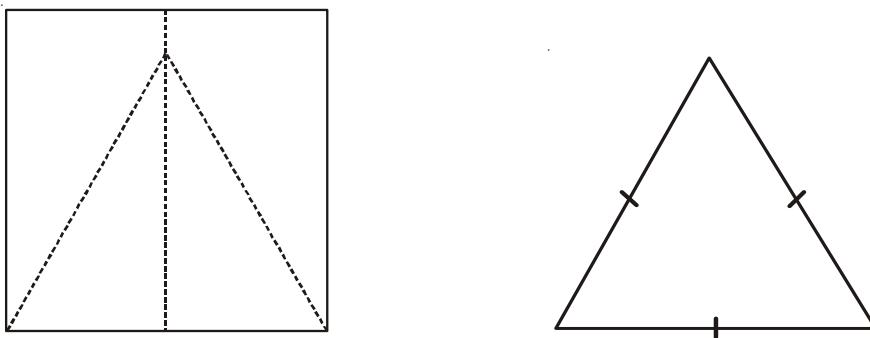
1.   Sum of the three angles in a triangle is 180⁰

$$\angle A + \angle B + \angle C = 180°$$

Cut a sheet of paper in the form a triangle. Mark the three angles as A, B, and C. Now cut the triangle into three pieces and observe that the vertices marked A, B, and C can be arranged in such a way that they form an angle of 180⁰, when put together. Hence the sum of the three angles in a triangle is 180⁰.
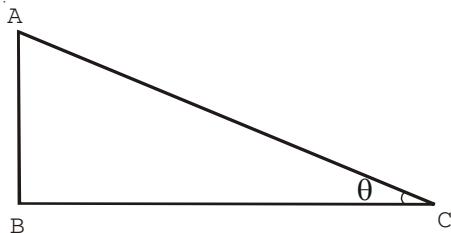
2.   Equilateral Triangle

Fold a sheet of paper vertically into two halves so that a crease is formed passing through the midpoints of two opposite sides. Now bring one lower corner of the paper to touch the crease already formed, ensuring that at the other lower end the
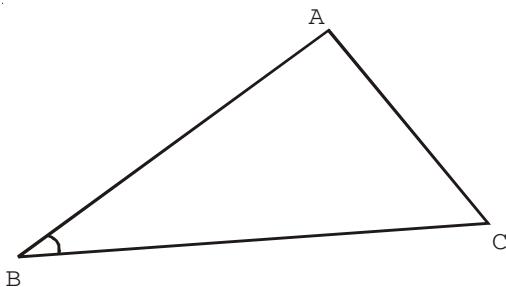
vertex is divided exactly into two halves. Mark the point at which the one lower end touches the crease formed. Now cut the paper twice with the marked point as one vertex and the lower two ends as the other two vertices. The resultant figure thus obtained is and equilateral triangle.

3. Right Angled Triangle
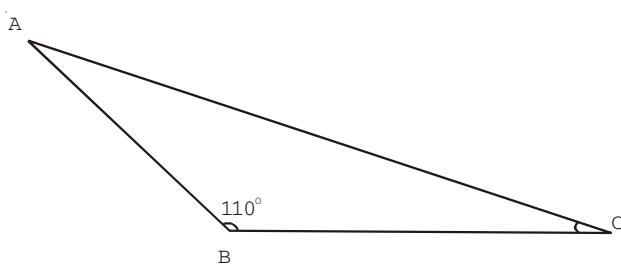
Take a neatly cut paper which forms right angles at its vertices. With the vertex containing the right angle intact cut the paper to form a triangle. The triangle formed is a right angled triangle.

4. Acute Angled Triangle
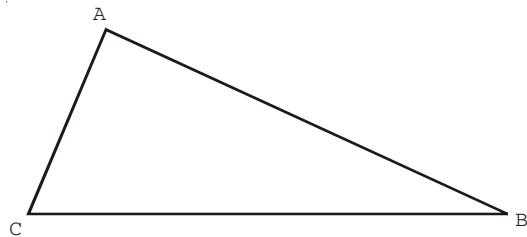
Take a neatly cut paper and mark an approximate point to denote its centre. Now cut off a triangle from the centre to any two vertices thus forming an acute angled triangle.

5. Obtuse Angled Triangle
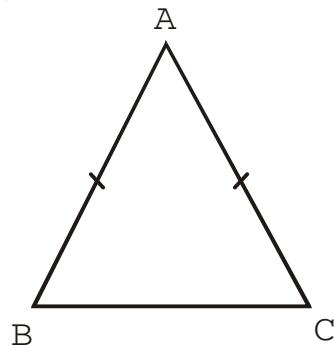
Cut a paper in the form of a triangle in such a way that one of the angles formed is an obtuse angle, that is, making an angle which measures more than 90^0. The triangle thus formed is an obtuse angled triangle.

6. Scalene Triangle
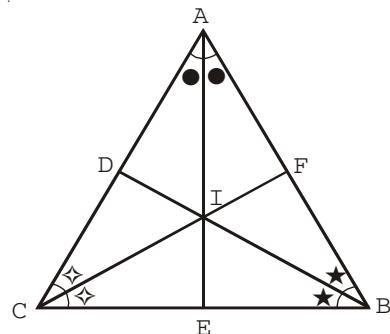
Take a neatly cut paper, choose any one vertex and cut the paper in such a way that no three sides are of equal length. The triangle thus formed is a scalene triangle.

7. Isosceles Triangle

Fold a neatly cut paper exactly into two halves. Mark a point on the impression thus created. Cut off a triangle from the point marked to any two vertices. The triangle thus formed is an isosceles triangle. ABC, DBC and EBC are all isosceles triangles.
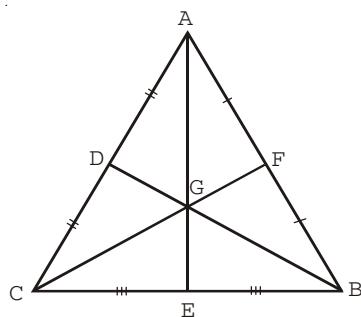
8. Incentre

Cut a sheet of paper in the form of a triangle. Fold the paper along the vertices to form three angle bisectors. Observe that the three angle bisectors pass through the same point, the incentre, denoted as I.

(**Note :** *Incentre is the point of concurrence of the angle bisectors of a triangle*)

9. **Centroid**
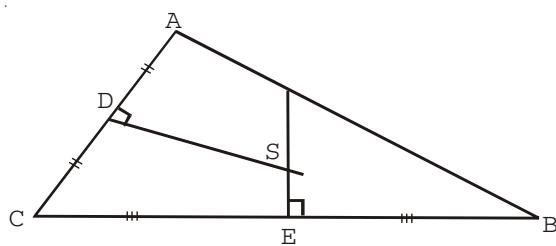
Cut a sheet of paper in the form of a triangle. Form the three medians by folding the triangle from the midpoint of a side to the opposite vertex. Observe that the three medians pass through the same point, the centroid, G.

(**Note :** *Median is the line joining the midpoint of a side to the opposite vertex. The point of concurrence of the medians of a triangle is the centroid, denoted as G.)*

10. **The centroid divides the medians in the ratio 2:1**
Form the centroid of a triangle in a sheet of paper. Cut a small strip of paper to the same length as that of the longer side of the median. Now fold the strip of paper into two and observe that it is of the same length as that of shorter side of the median. Thus the cetroid of a triangle divides the medians in the ratio 2:1.

11. **Circum Centre**
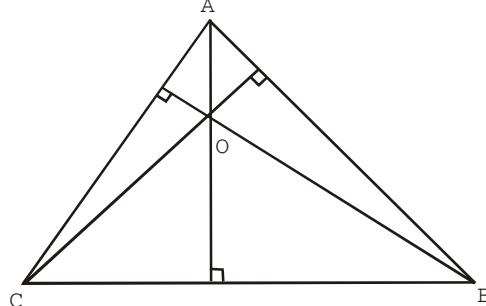
Cut a sheet of paper in the form of a triangle. Select a side and fold the paper to form a line along the midpoint of the side, the perpendicular bisector. Similarly form the other two perpendicular bisectors also. Observe that the three perpendicular bisectors pass through the same point, the circumcentre, S.

Note :

1. *Circumcentre is the point of concurrence of the perpendicular bisectors of a triangle.*
2. *In a right angle triangle the circumcentre divides the hypotenuse into two halves.*
3. *The circumcentre of an acute angle triangle is inside the triangle.*

12. Ortho Centre

Cut a sheet of paper in the form of a triangle. Select a side and fold the paper from the vertex to the side in such a way that the impression created and the side chosen are at right angles, which is an altitude. Similarly form the other two altitudes. Observe that all the three altitudes pass through the same point, the orthocentre, O.

Note :
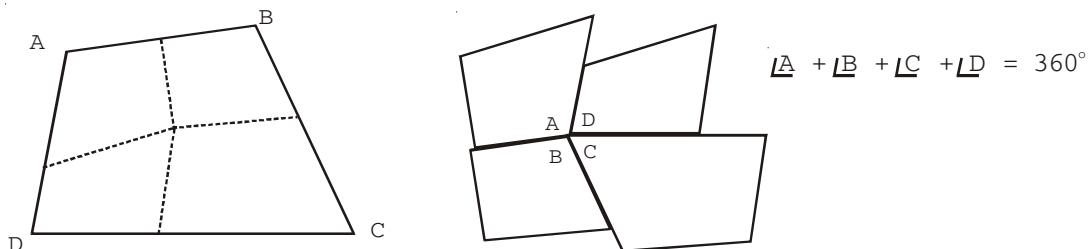
1. *The perpendicular drawn from one vertex to the base opposite to it is the altitude. The point of concurrence of the altitudes of a triangle is the orthocentre.*
2. *In a right triangle the orthocentre coincides with the vertex containing the angle 90°.*
3. *In an obtuse angled triangle the orthocentre is formed outside the triangle.*

Module 89

QUADRILATERALS
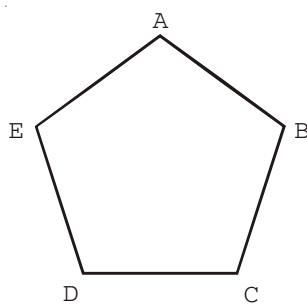
1. Sum of the angles of a quadrilateral is 360^0

$\angle A + \angle B + \angle C + \angle D = 360°$
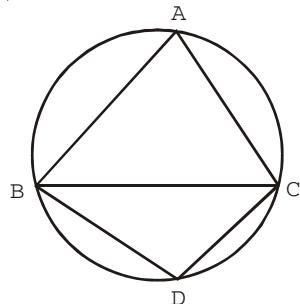
Cut a sheet of paper in the form of a quadrilateral. Mark the four angles of the vertices as A, B, C and D. Cut the quadrilateral into four pieces so that the angles A, B, C and D get separated into four pieces. Now arrange the four pieces adjacent to each other to form an angle of 360^0, revealing the fact that the angle formed in a quadrilateral is 360^0.

2. Pentagon through knot

Cut a sheet of paper in the form of a strip for a suitable length and width say $30cm \times 4$ cm. Insert one end of the paper into the other to form a knot and gently pull the paper on both ends so that a pentagon is formed. Now fold the sides so that the paper strip takes a fine pentagon shape.

3. Angle in a minor segment is obtuse and Angle in a major segment is acute


Cut a sheet of paper in the form of a circle. Fold the circle to form a chord thus forming a major segment and a minor segment. From the extremities of the chord fold the paper two times on either side of the chord to form two angles-one in the major segment and the other in the minor segment. Observe that the angle formed in the minor segment is always obtuse and the angle in the major segment is always acute.
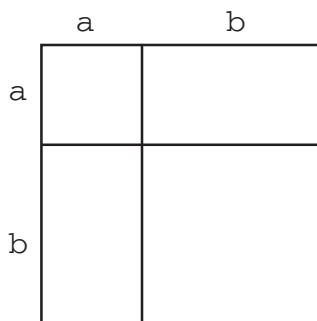
IDENTITIES

1. **(A+B)² = A² + 2AB + B²**

    Take a square sheet of paper. Fold it either vertically or horizontally so that the paper is divided into two parts one larger and the other smaller. Bring the vertex of the smaller part to coincide with the impression already created and mark the point where the vertex and the impression of the folding coincide. Fold the paper across that point to form a perpendicular line with the former. Now the paper is divided into four parts viz., two squares and two rectangles. Name the side of the two squares as 'a' and 'b'. Consequently the dimensions of the two equal rectangles will be a x b. Here the square with side (a+b), and hence the area $(a+b)^2$ is now divided into four parts, two squares with the area a^2 and b^2 respectively and two rectangles with area 'ab' each. This implies that $(a+b)^2 = a^2 + 2ab+b^2$.
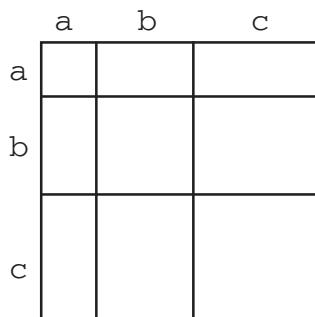

2. **(A+B+C)² = A² + B² + C² +2AB +2BC + 2CA**

    Cut a sheet of paper in the form of a square. Fold the paper say vertically to some suitable measurement. Now fold the paper horizontally so as to form two squares-one small and one big and two equal rectangles. Repeat the procedure again to divide the bigger square into two resulting in the formation of three squares of different sides

and 3 pairs of equal rectangles of dimensions corresponding to the side of each square. Let the side of the squares thus formed be a, b and c respectively and so the dimensions of the rectangles formed are a x b, b x c and c x a. Now observe that a single square has been divided into 3 squares of different dimensions and 3 pairs of equal rectangles. Thus the formation of the above said squares and rectangles reveal that $(A+B+C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$.

3. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} + \dfrac{1}{64} + \dfrac{1}{128} + \dfrac{1}{256} + \dfrac{1}{512} + \dfrac{1}{1024} + \ldots = 1$

Take a sheet of paper and fold it to form exactly two halves. Fold the one half again to form another two halves. Similarly repeat the procedure, say for instance, ten times so that the final folding gives a part which is $\dfrac{1}{1024}$ of the original paper, thus revealing the fact that $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} + \dfrac{1}{64} + \dfrac{1}{128} + \dfrac{1}{256} + \dfrac{1}{512} + \dfrac{1}{1024} + \ldots = 1$
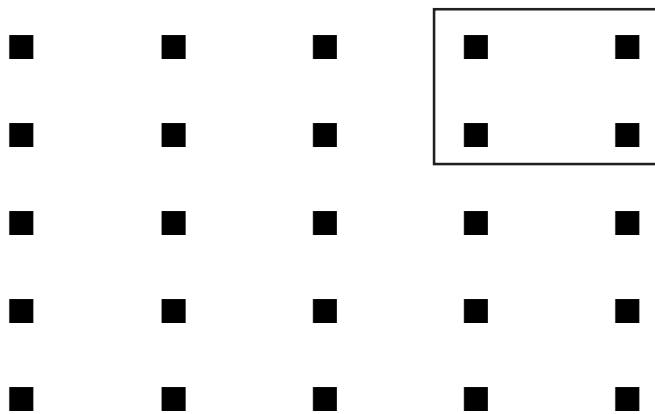
4. **Identity $A^2 - B^2 = (A+B)(A-B)$ through dots**
The identity can be illustrated through an example.
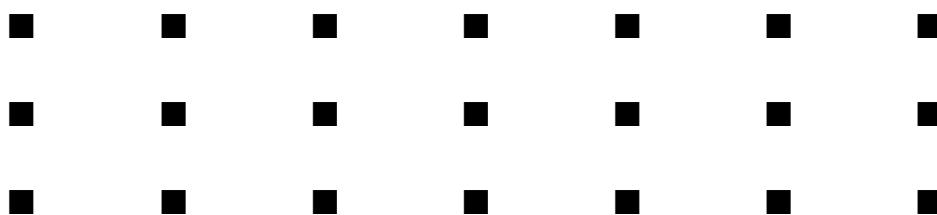
Let A = 5, B = 2

To prove, $5^2 - 2^2 = (5+2)(5-2)$

Construct the following dots in a sheet of paper.

■   ■   ■   ■   ■   Here $5^2 - 2^2 = 25 - 4 = 21$

■   ■   ■   ■   ■

■   ■   ■   ■   ■

■   ■   ■   ■   ■

■   ■   ■   ■   ■

Now rearrange the leftover dots barring the four dots as

■   ■   ■   ■   ■   ■   ■

■   ■   ■   ■   ■   ■   ■

■   ■   ■   ■   ■   ■   ■

Here, $5^2 - 2^2$ = (5+2) (5-2)

= (7) (3)

= 21   Therefore, $A^2 - B^2 = (A+B)(A - B)$

The illustrations given in this section are not exhaustive. As the title revels, it is a "Creative SEction" and therefore new ideas for paper folding may always emerge when the user aims for creative ways of teaching mathematical concepts to children with visual impairment.

We hope the learning of this package has helped you to become more confident in teaching mathematics to children with visual impairment.

References

1. Lancelot Hog Ben (1945). Mathematics For the Million: George Allen And Unwin Ltd, London

2. Jurgensen, Donnelly and Dolciani (1965). Modern Geometry: Houghton Mifflin Company, Connecticut.

3. Moise and Downs (1965). Geometry: Addison Wesley Publishing Company Inc, Massachussetts.

4. Mentor Books (1965). The Universal Encyclopedia of Mathematics: The New American Library of World Literature, New York 10019

5. Robert E.Eicholz, Phares G, O'Daffer, Merrill E.Shanks, Charles R.Fleenor (1967). School Mathematics Vol I and II: Addison Wesley Publishing Company, California

6. Vivian Shaw Groza, Susanne Shelley (1972). Pre Calculus Mathematics, Holt, Rinehart and Winston Inc., New York

7. James Lighthill (1978). Newer Uses of Mathematics, Penguin Books Ltd, England

8. Gail F, Burrill, Timothy D, K. Anold and Jerry J.Cummins (1993). Merrill Geometry (Application and Connection)

9. David Wells (1997). The Penguin Dictionary of Curious and Interesting Numbers: Penguin Books Ltd, England

10. Charles D. Miller, Vern E. Heeren, E. John Hornsby Jr (1998) Mathematical Ideas: Addison Wesley, Longman Inc, Massachusetts.

11. Albert B. Bennett Jr and L.Ted Nelson (2001). Mathematics for Elementary Teachers: Mc Graw Hill, Boston, Burr Ridge.

12. Angel and Porter (2001). A Survey of Mathematics with Application: Addison Wesley, Longman Inc, New York.

13. National Council of Educational Research and Training Text Books VI – XII (2004): NCERT, New Delhi.

14. Tamil Nadu Textbook Corporation Text Books I – X (2004). Government of Tamil Nadu, India.

15. Mani, M.N.G. (1992). Techniques of Teaching Blind Children. New Delhi : Sterling Publishers.

16. Mani, M.N.G. (1998). Amazing Abacus. Coimbatore : Sri Ramakrishna Mission Vidyalaya College of Education.

17. Mani, M.N.G. (1999). Braille. Coimbatore : Coimbatore : Sri Ramakrishna Mission Vidyalaya College of Education.